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MODULE - I

BASIC ANTENNA PARAMETERS

Radiation is the process of transmitting electromagnetic waves into space.

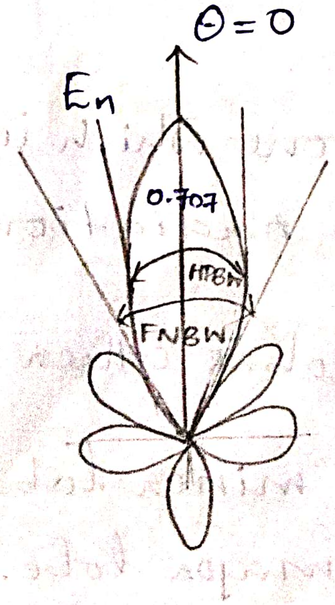
Antenna is a system to radiate or receive electromagnetic signals.

Parameters of Antenna.

1. Radiation Pattern.

An antenna radiation pattern is defined as a mathematical function or graphical representation of antenna parameters (radiation properties) as a function of space coordinates.

→ Amplitude field Pattern.



HPBW - Half Power Beam Width.

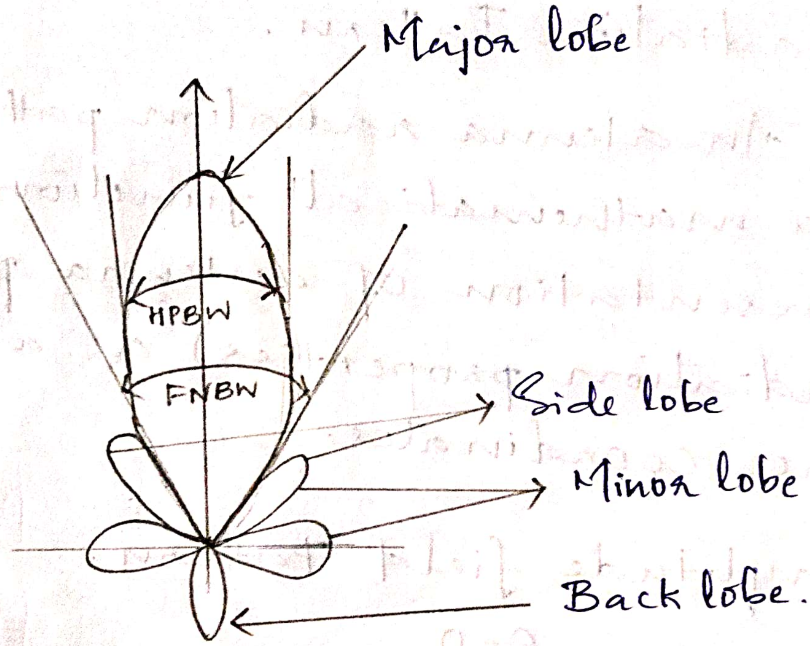
FNBW - First Null Beam width.

The trace of received electric/magnetic field at a constant radius is called amplitude field pattern.

→ Amplitude power pattern:

It is a Spatial Variation of power density along a constant radius.

Radiation Pattern lobes.



* Major lobe: It is the one which is in the direction of maximum radiation.

* Minor lobe: All the lobes other than major lobe

* Side lobe: It is the minor lobe which is adjacent to major lobe.

* Back lobe: It is the radiation that is opposite to major lobe.

2. Poyinting Vector [Radiation Power Density]

It is used to describe the power associated with electromagnetic wave and is defined as the electromagnetic energy crossing through unit area in one second.

$$W = E \times H \quad \text{--- (1)}$$

E = Instantaneous Electric field intensity

H = Instantaneous Magnetic field intensity.

$$E = Re\left[E\, e^{j\omega t}\right] = \frac{E e^{j\omega t} + E^{*} e^{-j\omega t}}{2}$$

$$H = Re\left[H\, e^{j\omega t}\right] = \frac{H e^{j\omega t} + H^{*} e^{-j\omega t}}{2}$$

$$W = (E \times H) = \left[\frac{E e^{j\omega t} + E^{*} e^{-j\omega t}}{2}\right] \times \left[\frac{H e^{j\omega t} + H^{*} e^{-j\omega t}}{2}\right]$$

$$= \frac{1}{4}\left[(E \times H) e^{2j\omega t} + (E \times H^{*}) + (E^{*} \times H) + (E^{*} \times H^{*}) e^{-2j\omega t}\right]$$

$$= \frac{1}{2}\left[\frac{(E \times H) e^{2j\omega t} + (E^{*} \times H^{*}) e^{-2j\omega t}}{2} + \frac{(E \times H^{*}) + (E^{*} \times H)}{2}\right]$$

$$W = \frac{1}{2}\left[Re\left[E \times H\right] e^{2j\omega t} + Re\left[E \times H^{*}\right]\right]$$

Time average poyinting Vector, $W_{av} = \frac{1}{2} Re(E \times H^{*})$

3. Radiation Intensity [U] / [φ]

It is defined as a power radiated from an antenna per unit solid angle. It is a far field parameter and can be obtained by multiplying radiation density by square of the distance.

$$\boxed{U = r^2 \cdot W_{rad}}$$

U = Radiation Intensity in Watts per unit solid angle.

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Radiation intensity can also express as a function of angles

$$U(\theta, \phi) = \phi(\theta, \phi) = \frac{r^2 E^2}{\eta_0} \quad W/Sr$$

P_{rad} = Power radiated

η_0 = free space impedance

$$\eta_0 = 120\pi$$

$$P_{rad} = \oiint U \, d\Omega$$

$d\Omega = \sin\theta \, d\theta \, d\phi$

$$= \oiint U \sin\theta \, d\theta \, d\phi$$

U_0 = radiation intensity of isotropic angle

$$P_{rad} = U_0 \oiint d\Omega$$

$$= U_0 \, 4\pi$$

$$U_0 = \frac{P_{rad}}{4\pi}$$

4. Directive Gain. (G_d)

Directive gain in a given direction is defined as the ratio of Radiation Intensity in that direction to the average radiated power.

$$G_d = \frac{U(\theta,\phi)}{P_{rad}/4\pi} = \frac{4\pi U(\theta,\phi)}{P_{rad}} = \frac{U(\theta,\phi)}{U_0}$$

Power Gain (G_p)

It is the ratio of power density radiated in a particular direction to that radiated in the same direction by an isotropic antenna for the same i/p power at the same distance.

$$G_p(dB) = 10 \log (P_1/P_2)$$

where P_1 is the power density by the directive antenna.

P_2 is the power density of isotrophic antenna.

Directivity (D)

It denotes the maximum directive gain.

$$\boxed{D = \left| \frac{U}{U_0} \right|_{max.}}$$

5. Radiation Efficiency (η) [Antenna Efficiency]

It is defined as the ratio of power radiated to the total power supplied to the antenna ie;

$$\boxed{\eta = \frac{P_r}{P_t} = \frac{P_r}{P_r + P_{Loss}}}$$

R_r = radiation resistance

R_L = Loss resistance

$$\eta = \frac{I^2 R_r}{I^2 (R_r + R_L)} = \frac{R_r}{R_r + R_L}$$

$$\eta = \frac{P_r}{P_t} = \frac{P_r}{P_t} \cdot \frac{4\pi \phi(\theta,\phi)}{4\pi \phi(\theta,\phi)}$$

$$= \frac{4\pi \phi(\theta,\phi)}{P_t} \cdot \frac{P_r}{4\pi \phi(\theta,\phi)}$$

$$= G_p \times \frac{1}{G_d}$$

$$\therefore \boxed{\eta = \frac{G_p}{G_d}}$$

$$\boxed{G_p = \eta G_d}$$

6. Radiation Resistance.

The power at a point where it is fed to the antenna experience a fictitious resistance which is called radiation resistance and

It is defined as the resistance which when substituted in series with the antenna will consume the same power as it is actually radiated.

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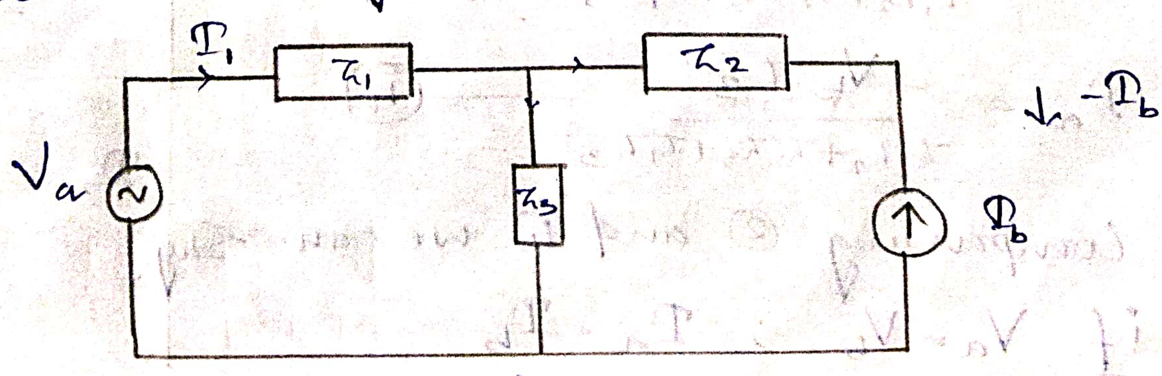
Reciprocity theorem:

If an emf is applied to the terminals of an antenna number 1 and the current measured at the terminals of antenna number 2, then an equal current both in amplitude and phase will be obtained at the terminal of antenna no. 1 if the same emf is applied to the terminals of antenna no. 2.

Inorder to prove the reciprocity theorem of antennas, let the antennas and the space between them be replaced with a network of linear bilateral and passive impedances. Since any four terminal network can be replaced with a T junction the antenna arrangement can be replaced with the n/w shown in figure:



By using current division rule,

$$-I_b = I_1 \frac{Z_3}{Z_3+Z_2}$$

$$I_b = -I_1 \frac{Z_3}{Z_3+Z_2} \quad \text{——— } ①$$

$$I_1 = \frac{V_a}{(Z_2 \| Z_3)+Z_1}$$

$$= \frac{V_a}{\frac{Z_2 Z_3}{Z_2+Z_3}+Z_1}$$

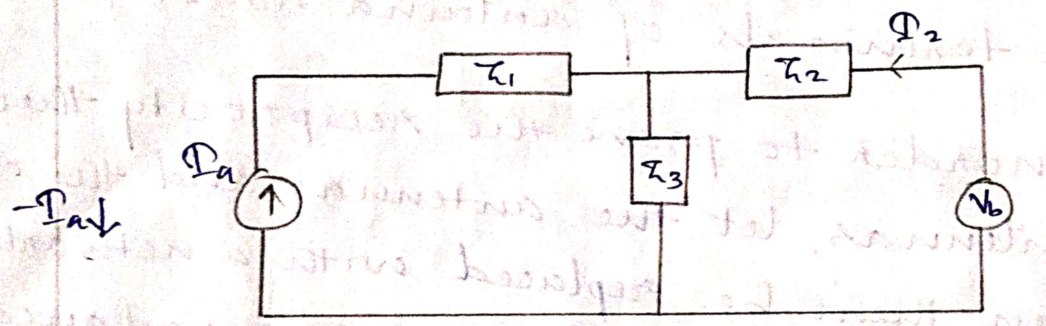
$$= \frac{V_a(Z_2+Z_3)}{Z_2Z_3+Z_1(Z_2+Z_3)}$$

$$I_1 = \frac{V_a(Z_2+Z_3)}{Z_2Z_3+Z_1Z_2+Z_1Z_3}$$

Substitute I_1 in eqn ①

$$I_b = \frac{-V_a\cancel{(Z_2+Z_3)}}{Z_1Z_2+Z_2Z_3+Z_1Z_3} \times \frac{Z_3}{\cancel{Z_3+Z_2}}$$

$$I_b = \frac{-V_a Z_3}{Z_1Z_2+Z_2Z_3+Z_1Z_3} \quad \text{——— } ②$$



$$-I_a = I_2 \times \frac{Z_3}{Z_1+Z_3} \quad \text{——— } ③$$

$$I_2 = \frac{V_b}{(Z_1 \| Z_3)+Z_2}$$

$$= \frac{V_b}{\frac{Z_1Z_3}{Z_1+Z_3}+Z_2}$$

$$= \frac{V_b(Z_1+Z_3)}{Z_1Z_3+Z_2(Z_1+Z_3)}$$

$$I_2 = \frac{V_b(Z_1+Z_3)}{Z_1Z_2+Z_2Z_3+Z_1Z_3}$$

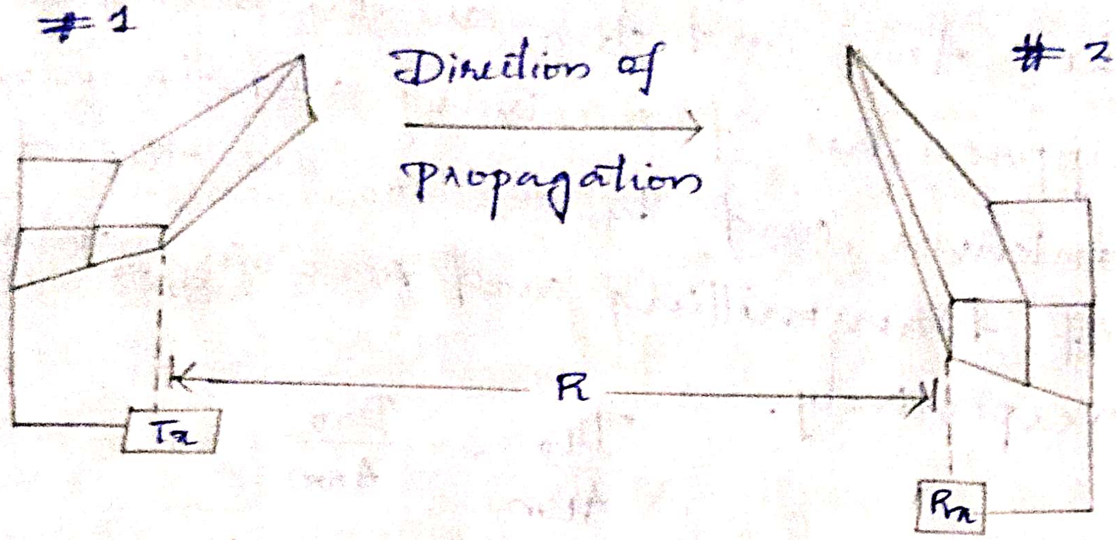
Substitute I_2 in eqn ③

$$I_a = \frac{-V_b\cancel{(Z_1+Z_3)}}{Z_1Z_2+Z_2Z_3+Z_1Z_3} \times \frac{Z_3}{\cancel{Z_1+Z_3}}$$

$$I_a = \frac{-V_b Z_3}{Z_1Z_2+Z_2Z_3+Z_1Z_3} \quad \text{——— } ④$$

Comparing ② and ④ we can say,
if $V_a = V_b$; $I_a = I_b$
Proving reciprocity theorem

Effective area / Effective Aperture.



Here antenna 1 is used as Transmitter and antenna 2 is used as receiver. The effective area and directivities of each are designated as A_t A_r, D_t and D_r. The radiation power density of the transmitter antenna is

$$W_t = \frac{P_t D_t}{4\pi R^2}$$

The power received by the antenna and transfered to the load would be,

$$P_r = W_t \times A_r = \frac{P_t D_t}{4\pi R^2} \cdot A_r$$

$$D_t A_r = \frac{P_r}{P_t} 4\pi R^2 \quad \text{——— (1)}$$

If antenna 2 is used as transmitter and 1 as receiver then,

$$P_r = P_t \,, \quad D_r A_t = \frac{P_r}{P_t} 4\pi R^2 \quad \text{——— (2)}$$

By comparing (1) and (2)

$$D_t A_r = D_r A_t$$

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$$\frac{D_t}{A_t} = \frac{D_r}{A_r}$$

Let A_{rm} and A_{tm} are the maximum effective approaches of receiving and transmitting Antenna and D_{to} & D_{ro} are the directivities of transmitting and receiving antennas respectively.

$$\frac{D_{to}}{A_{tm}} = \frac{D_{ro}}{A_{rm}}$$

Then assume the transmitting antenna be an isotropic one then its ~~dir~~ directivity $D_{to} = 1$

$$\frac{1}{A_{tm}} = \frac{D_{ro}}{A_{rm}}$$

$$A_{rm} = A_{tm} D_{ro}$$

$$A_{rm} = \frac{\lambda^2}{4\pi} \cdot D_{ro}$$

[Effective Aperture = $\lambda^2/4\pi$ Directivity]

Generally, $\boxed{A_e = \frac{\lambda^2}{4\pi} D}$

Helm Holtz Theorem.

It states that any well defined vector field can be decomposed into the sum of a longitudinal vector field (diverging non-curling, irrotational) and a transverse vector field (solenoidal, curling, rotational, non-diverging).

$$\boxed{F = F_l + F_t}$$

The divergence and curl can be thought of as orthogonal operators as their product equal to zero ie; $\nabla \times \nabla\phi = 0$

$$\nabla \cdot (\nabla \times A) = 0$$

Proof:

Let $F(x)$ be a general vector field where x is the 3D observation point vector. Expand F into an integral form using dirac delta function.

$$F(x) = \int_V F(x')\,\delta(x - x')\,dx' \quad \text{—— (1)}$$

Dirac function:

$$\boxed{\delta(x - x') = -\frac{1}{4\pi}\nabla^2\left(\frac{1}{|x - x'|}\right)}$$

Substitute in eqn (1);

$$f(x) = \int_V f(x') \cdot -\frac{1}{4\pi} \nabla^2 \left(\frac{1}{|x-x'|} \right) dx'$$

$$= -\frac{1}{4\pi} \nabla^2 \int_V \frac{f(x')}{|x-x'|} dx'$$

Use the identity; $\boxed{\nabla^2 A = \nabla \cdot (\nabla \cdot A) - \nabla \times (\nabla \times A)}$

$$\therefore f(x) = \frac{-1}{4\pi} \left[\nabla \cdot \left(\nabla \cdot \int_V \frac{F(x')}{|x-x'|} dx' \right) - \nabla \times \left(\nabla \times \int_V \frac{F(x')}{|x-x'|} dx' \right) \right]$$

$$f(x) = \underbrace{-\frac{1}{4\pi} \left[\nabla \cdot \left(\nabla \cdot \int_V \frac{F(x')}{|x-x'|} dx' \right) \right]}_{f_l} + \underbrace{\frac{1}{4\pi} \left[\nabla \times \left(\nabla \times \int_V \frac{F(x')}{|x-x'|} dx' \right) \right]}_{f_t}$$

Identify the first term as a longitudinal term and second term as a transverse term ie;

$$F = F_l + F_t$$

Taking the curl of F_l shows it is longitudinal and divergence of F_t shows it is transverse.

$$\nabla \times F_l = \nabla \times \left[\frac{-\nabla}{4\pi} \nabla \cdot \int \frac{F(x')}{|x-x'|} dx' \right]$$

$\nabla \times F_l = 0$ because curl of gradient $= 0$.

$$\nabla \cdot F_t = \nabla \cdot \frac{1}{4\pi} \nabla \times \left[\nabla \times \int \frac{f(x')}{|x - x'|} dx' \right]$$

ie; $\nabla \cdot F_t = 0$ Since divergence of curl is always zero.

This means one term is purely solenoidal and the other is purely diverging and their sum gives a general vector field. This is Helmholtz decomposition theorem.

Q. Derive the radiational resistance of a Half wave dipole antenna.

To find the radiation resistance, the poyinting vector is integrated over a large sphere to compute the power, then equate to $I^2 R$ to get the radiational resistance.

Power radiated $P_r = \oiint S_r \, ds$

$$P_r = \oiint \tfrac{1}{2} |H_\phi|^2 \eta \cdot r^2 \sin\theta \, d\theta \, d\phi$$

We know $|H_\phi| = \frac{I_m}{2\pi r} \frac{\cos(\pi/2 \cos\theta)}{\sin\theta}$

Substituting,

$$P_r = \int_0^{2\pi} \int_0^{\pi} \tfrac{1}{2} \left[\frac{I_m}{2\pi r} \frac{\cos(\pi/2 \cos\theta)}{\sin\theta} \right]^2 \cdot \eta r^2 \sin\theta \, d\theta \, d\phi$$

Poynting Vector

$S_r = \tfrac{1}{2} Re [E_\theta \cdot H_\phi^*]$

$E/H = \eta$

$E = \eta H$

$= \tfrac{1}{2} Re [\eta H \cdot H_\phi^*]$

$S_r = \tfrac{1}{2} \cancel{Re} |H_\phi|^2 \eta$

$S_r = \tfrac{1}{2} |H_\phi|^2 \eta$

$ds = r^2 \sin\theta \, d\theta \, d\phi$

Small area along a sphere.

$$P_r = \int_0^{2\pi}\int_0^{\pi} \frac{1}{2}\left[\frac{I_m}{2\pi r}\cdot\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\right]^2 \eta r^2 \sin\theta \, d\theta \, d\phi$$

$$= 2\pi \int_0^{\pi} \frac{1}{2}\frac{I_m^2}{4\pi^2 r^2}\frac{\cos^2(\pi/2\cos\theta)}{\sin^2\theta} \, 120\pi \times r^2 \sin\theta \, d\theta \qquad \int_0^{2\pi} d\phi = [\phi]_0^{2\pi} = 2\pi.$$

$$= 30 I_m^2 \int_0^{\pi} \frac{\cos^2(\pi/2\cos\theta)}{\sin\theta} d\theta \qquad \eta = 120\pi \qquad \int_0^{\pi} = 2\int_0^{\pi/2}$$

$$= 60 I_m^2 \int_0^{\pi/2} \frac{\cos^2(\pi/2\cos\theta)}{\sin\theta} d\theta$$

2/8/24 P_r is integrated using trapezoidal rule and the solution of this integral value is 0.6096

$$\therefore P_r = 60 I_m^2 \times 0.6096$$

$$P_r = 36.5 I_m^2 \quad \text{—— (1)}$$

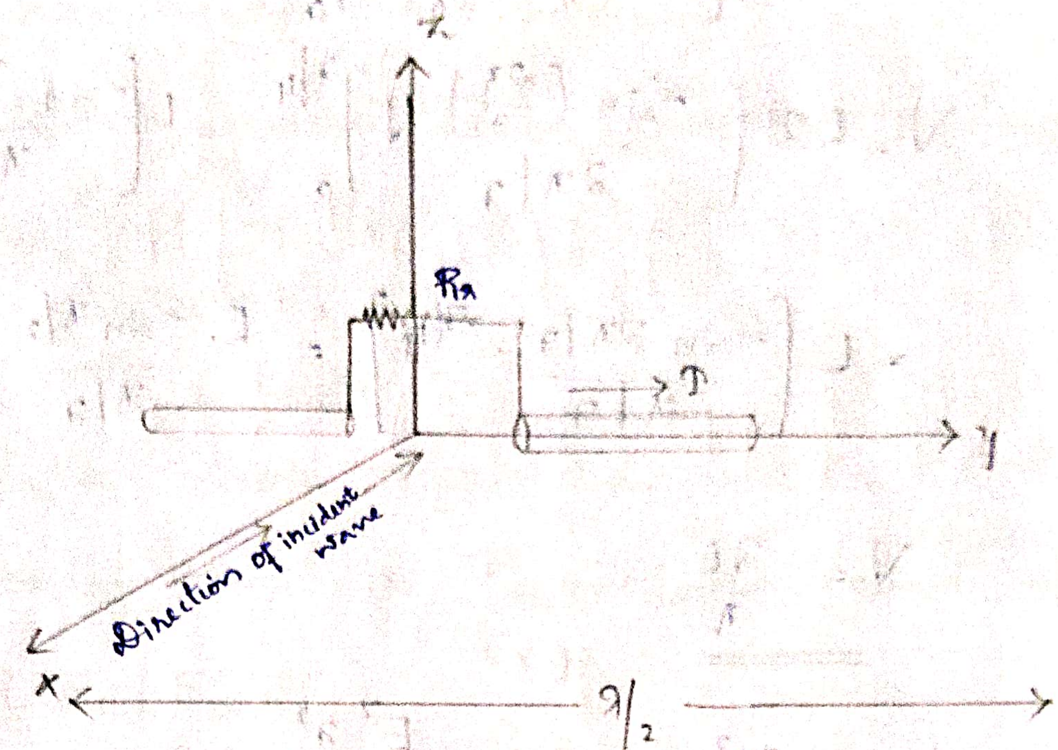
$$P = I^2 R$$
$$= \left(\frac{I_m}{\sqrt{2}}\right)^2 R = \frac{I_m^2}{2} R \quad \text{—— (2)}$$

Equate (1) and (2)

$$\frac{I_m^2}{2} R = 36.5 I_m^2$$

$$\therefore R = 73\,\Omega$$

Directivity of Half Wave ($\lambda/2$) Antenna.



Inorder to compute the directivity of half wave dipole antenna, first compute the effective aperture, then using the equation,

$$D = \frac{4\pi A_e}{\lambda^2}$$ we can find the

directivity.

We know effective aperture, $A_e = \frac{V^2}{4 S_r R_r}$

where V is the induced Voltage drop

R_r is the rational resistance of $\lambda/2$ antenna

S_r is the poyinting vector $= E^2/\eta$

For a small element dy, the voltage induced dv for $\lambda/2$ antenna is $dv = E\,dy \cos\left(\frac{2\pi}{\lambda}\right)y$ $E\cos\left(\frac{2\pi}{\lambda}\right)y\,dy$

The induce voltage $V = E2\int_0^{\lambda/4} \cos\left(\frac{2\pi}{\lambda}\right)y\,dy$

$$= 2\left[\sin\left(\frac{2\pi}{\lambda}\times\frac{\lambda}{2\pi}\right)\cdot y\right]_0^{\lambda/4}$$

$$V = E2\left[\frac{\sin\left(\frac{2\pi}{\lambda}\right)}{2\pi/\lambda}\, y\right]_0^{\lambda/4} = E\left[\frac{\sin(2\pi/\lambda)}{\pi/\lambda}\, y\right]_0^{\pi/4}$$

$$= E\left[\frac{\sin 2\pi/\lambda \,.\, \lambda/4}{\pi/\lambda}\right] = E.\frac{\sin \pi/2}{\pi/\lambda}$$

$$V = \frac{\lambda E}{\pi}$$

$\eta = 120\pi$

$$A_e = \frac{V^2}{4 S_a R_r} = \frac{\frac{E^2\lambda^2}{\pi^2}}{4\times\frac{E^2}{\eta}\times 73} = \frac{\lambda^2\times 120\pi}{4\pi^2\times 73}$$

$$A_e = 0.13\lambda^2$$

$$D = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi\times 0.13\lambda^2}{\lambda^2} = 1.63$$

$$\boxed{D = 1.63}$$

5/8/24 Derive the radiation resistance of a short electric dipole.

The radiation resistance of a short dipole is determined by following the steps

1. The power radiated is obtained by integrating poyinting vector of far field over a large sphere.

2. This power radiated is equated to I^2R to find the radiation resistance.

Poying Vector have three components:

$$S_r = E_\theta H_\phi$$
$$S_\theta = -E_r H_\phi$$
$$S_\phi = E_\theta H_r$$

But for a Short dipole H_r and $E_r = 0$

$\therefore$ S_r only exist $\Rightarrow$ $S_r = E_\theta H_\phi$ and also

$$S_r = \frac{1}{2} Re\left[E_\theta H_\phi^*\right] \qquad E_\theta = H_\phi \eta$$
$$= \frac{1}{2} Re\left[\eta H_\phi \cdot H_\phi^*\right]$$
$$= \frac{1}{2} |H_\phi|^2 \eta$$

Then integrate S_r over the Sphere to get the power radiated.

$$P_r = \iint S_r \cdot ds \qquad ds = r^2 \sin\theta \, d\theta \, d\phi$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{1}{2} \eta |H_\phi|^2 ds \qquad H_\phi = \frac{I_0 L}{4\pi} \sin\theta \left(\frac{\omega}{Cr}\right)$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{1}{2} \eta \left[\frac{I_0 L}{4\pi} \sin\theta \left(\frac{\omega}{Cr}\right)\right]^2 r^2 \sin\theta \, d\theta \, d\phi$$

$$P_r = \frac{1}{2} \eta \left(\frac{I_0 L}{4\pi}\right)^2 \left(\omega/c\right)^2 \int_0^{2\pi}\int_0^{\pi} \sin^3\theta \, d\theta \, d\phi \quad \text{——} (1)$$

By using Walli's equation,

$$\int_0^{\pi/2} \sin^n\theta \, d\theta = \frac{n-1}{n} \quad \text{for } n = \text{odd}$$
$$= \pi/2 \, \frac{n-1}{n} \quad \text{for } n = \text{even}$$

$$2\int_0^{\pi/2} \sin^3\theta \, d\theta = 2 \times \frac{3-1}{3} = \frac{4}{3}$$

$\therefore$ Equation ① becomes,

$$P_r = \frac{1}{2}\eta \left(\frac{I_0 L}{4\pi}\right)^2 \left(\omega/c\right)^2 \frac{4}{3} \cdot \int_0^{2\pi} d\phi$$

$$= \frac{1}{2}\eta \left(\frac{I_0 L}{4\pi}\right)^2 \left(\omega/c\right)^2 \frac{4}{3} \times 2\pi$$

$$= \frac{1}{\cancel{2}} \times \cancel{120\pi}\,\frac{(I_0 L)^2}{\cancel{16\pi^2}} \times \frac{\cancel{4\pi^2}}{\lambda^2} \times \frac{8\pi}{\cancel{3}}$$

$$= \frac{40\pi^2 (I_0 L)^2}{\lambda^2} \quad \text{—— ②}$$

$\eta = 120\pi$

$\omega = 2\pi f$

$\lambda = c/f$

$c = \lambda f$

$\omega^2 = 4\pi^2 f^2$

$c^2 = \lambda^2 f^2$

$\therefore \frac{\omega^2}{c^2} = \frac{4\pi^2 f^2}{\lambda^2 f^2} = \frac{4\pi^2}{\lambda^2}$

We know Power $P = I_0^2 R$

$$P = \left[\frac{I_0}{\sqrt{2}}\right]^2 R = \frac{I_0^2}{2} R \quad \text{—— ③}$$

② = ③

$$\Rightarrow \frac{40\pi^2 I_0^2 L^2}{\lambda^2} = \frac{I_0^2}{2} R$$

$$= 40\pi^2 I_0^2 \left(\frac{L^2}{\lambda^2}\right) = \frac{I_0^2 R}{2}$$

L = length of dipole

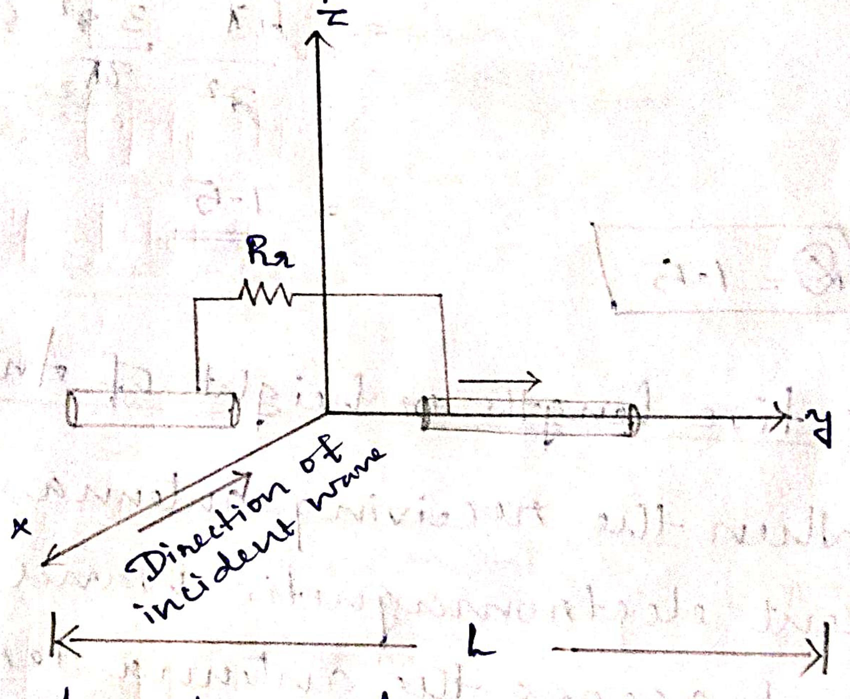
λ = wavelength

$$\boxed{R_r = 80\pi^2 \left(\frac{L}{\lambda}\right)^2}$$

Derive the Directivity of Short dipole Antenna

Inorder to compute the directivity, first compute the effective aperture, then using the formula $D = \frac{4\pi A_e}{\lambda^2}$ find directivity.

Consider a short dipole with uniform current induced by an incident wave. Let the dipole having length L and current I are shown in figure:



Effective Aperture $A_e = \frac{V^2}{4 S R_r}$ where

V = Induced Voltage $\qquad R_r = 80\pi^2 (L/\lambda)^2$

S = Poyinting Vector $= \frac{E^2}{\eta}$ $\qquad \because V = E \cdot L$

$$\therefore A_e = \frac{E^2 L^2}{4 \times \frac{E^2}{\eta} \times 80\pi^2 \left(\frac{L}{\lambda}\right)^2}$$

$$A_e = \frac{E^2 L^2}{4 \times \frac{E^2}{\eta} \times 80\pi^2 \frac{L^2}{\lambda^2}}$$ $\eta = 120\pi$

$$= \frac{120\pi \lambda^2}{4 \times 80\pi^2} = \frac{3}{8\pi}\lambda^2$$

$$A_e = \frac{3}{8\pi}\lambda^2$$

$\therefore$ Directivity, $D = \frac{4\pi}{\lambda^2} \times A_e$

$$= \frac{4\pi}{\lambda^2} \frac{3\lambda^2}{8\pi} = 3/2$$

$$= 1.5$$

$$\boxed{D = 1.5}$$

6/8/24 **Effective length or height of Antenna**

When the receiving antenna intersects incident electromagnetic waves a Voltage is induced across the antenna terminal. The effective height is defined as the ratio of open circuit terminal Voltage to the incident electric field strength in the direction of antenna polarisation.

$$h_e = \frac{V_{oc}}{E} \text{ meters}$$

V_{oc} = Open-ckt Vge

E = Electric field Strength

Dual characteristics of Antenna

The duality of an antenna specifies a circuit device on one hand and a space device on other hand.

Circuit quantities.

Space quantities

Antenna Impedance (Z_A)

Radiation resistance (R_r)

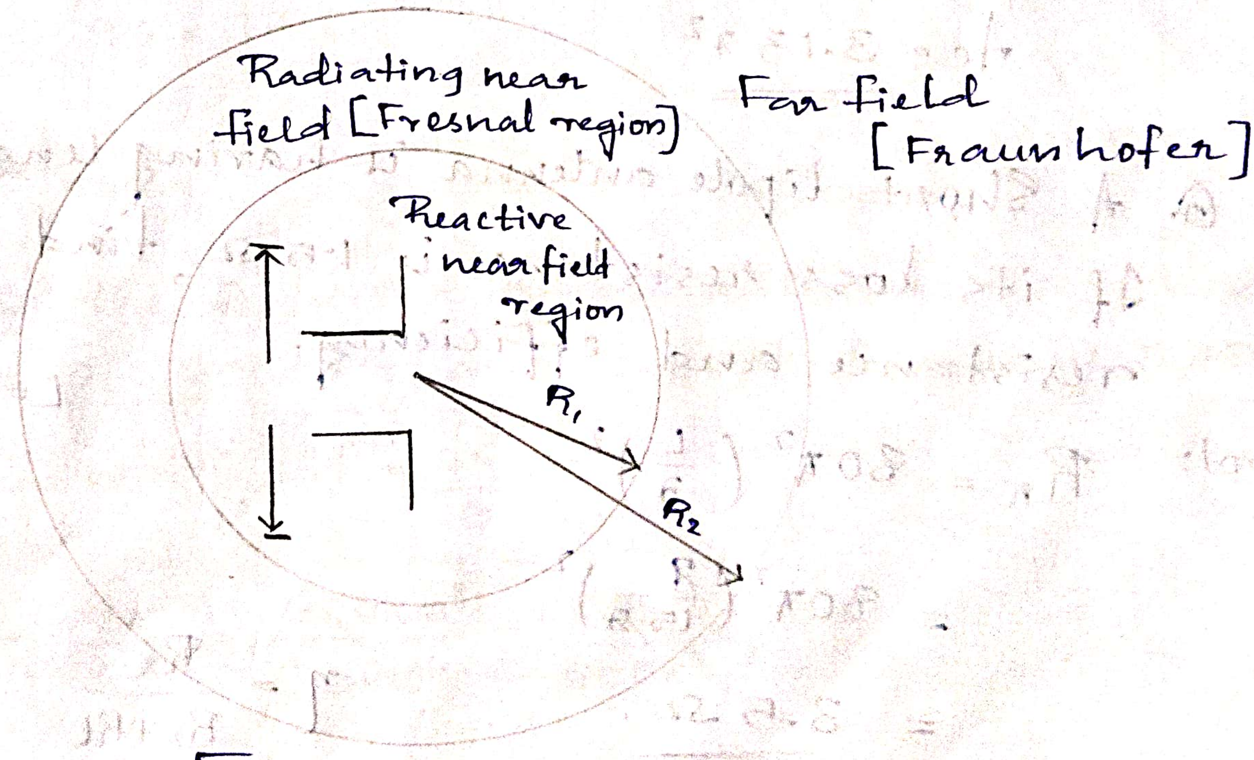
Antenna temperature (T_A)

Current Distribution

* Field Pattern $E_\theta(\theta,\phi)$, $E_\phi(\theta,\phi)$
* Polarization → (LP, CP, EP)
* Power Pattern
* Beam area
* Directivity
* Gain
* Effective Aperture

Define
- Antenna temperature
- Beam Area.

Antenna Field Zones:



$$R_1 = 0.62\sqrt{\frac{D^3}{\lambda}}$$

$$R_2 = \frac{2D^2}{\lambda}$$

Q. Find the maximum effective aperture of a beam antenna having half power beam width of 30° and 35° in perpendicular planes intersecting in the beam axis.

$$\boxed{D = \frac{41257}{\theta_E \times \theta_H}}$$

Sol: $\theta_E = 30°$ $\theta_H = 35°$

$$D = \frac{41257}{30 \times 35} = 39.292$$

$$= \underline{\underline{39.3}}$$

$$\therefore A_e = \frac{\lambda^2}{4\pi} \times D = \frac{\lambda^2}{4\pi} \times 39.3$$

$$= \underline{\underline{3.127\lambda^2}}$$

$$\therefore A_e = \underline{\underline{3.13\lambda^2}}$$

Q. A Short dipole antenna is having length $\lambda/15$. If its loss resistance is 1.5 Ω, find radiation resistance and efficiency.

$L = \frac{\lambda}{15}$

Sol: $$R_r = 80\pi^2 \left(\frac{L}{\lambda}\right)^2$$

$$= 80\pi^2 \left(\frac{\cancel{\lambda}}{15\cancel{\lambda}}\right)^2$$

$$= \underline{\underline{3.5\,\Omega}}.$$

$$\eta = \frac{R_r}{R_r + R_l} = \frac{3.5}{3.5+1.5}$$

$$= 0.7$$

$$\therefore \eta = \underline{\underline{70\%}}$$

Q. An atenna has radiational resistance 72 Ω, loss resistance 8 Ω, $G_p = 12\,dB$. Find efficiency and directivity.

$R_r = 72\Omega$

$R_l = 8\Omega$

Sol: Efficiency $\eta = \frac{R_r}{R_r + R_l}$

$= \frac{72}{72+8} = 0.9$

$\eta = 90\%$

$\boxed{\eta = \frac{G_P}{G_d}}$

$G_p = 12\,dB$

$10 \log G_p = 12\,dB$

$\therefore G_p = 15.84$

$\log(G_p) = 1.2$

$\therefore G_p = \text{antilog}(1.2)$

$\therefore G_d = \frac{G_p}{\eta} = \frac{\cancel{0.9\times}15.84}{0.9} =$

$G_d = 17.6.$

$|G_d|_{max} = D = 17.6.$

Q. Find the Q factor of an antenna if it has a B.W of 600KHz and cut off frequency 30MHz.

Bandwidth, $\boxed{\Delta f = \frac{f_r}{Q}}$

Sol: $Q = \frac{f_r}{\Delta f} = \cancel{\frac{600\times10^3}{30\times10^6}} = \frac{30\times10^6}{600\times10^3}$

$Q = 50$

Q. Calculate the radiational resistance of an antenna which draws 15 A current and radiates a 5KW.

Soli: $P_r = I^2_{rms} \times R_r$

$$R_r = \frac{P_r}{I^2_{rms}} = \frac{5 \times 10^3}{15^2} = 22.22\Omega$$

Antenna Temperature:

Antenna temperature is a measure of the noise power received by an antenna. It is defined as the temperature of a matched load that would produce the same amount of noise power at the antenna o/p as the actual antenna receives from the surrounding environment.

Beam Area:

It describes the angular spread of an antenna's ~~theory~~ radiation pattern, defining the region where the radiated energy is concentrated.

Beam Area = Beam Width x Beam Width

3/24

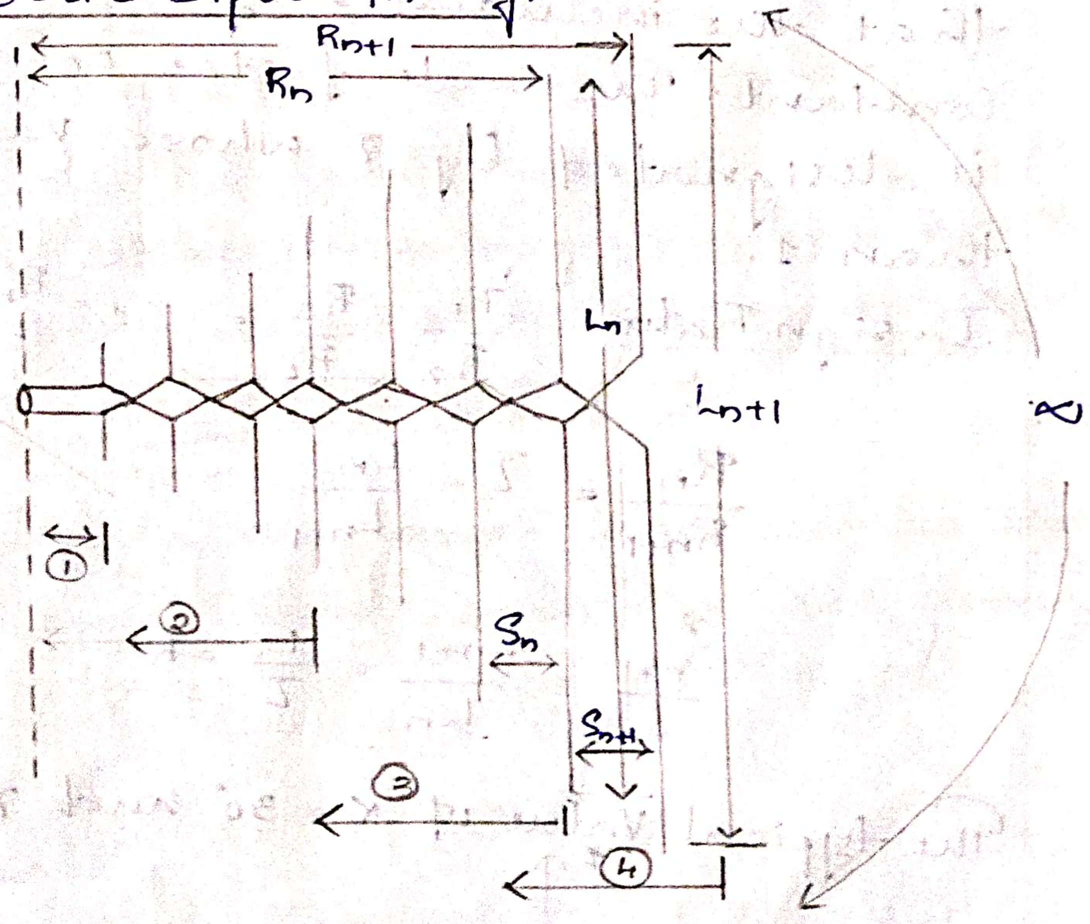
Module - 2

BROADBAND ANTENNAS

Frequency Independent Antennas.

In a frequency independent antenna, inorder to make frequency independent, the antenna should expand or contract in proportion to the wavelength or if the antenna structure is not mechanically adjustable, then the size of radiational region should be proportional to lambda (λ).

Log Periodic Dipole Array.



1, 2 } Capacitive , $L < \lambda/2$

3 - Resistive , $L = \lambda/2$

4 - Inductive , $L > \lambda/2$

An LPDA is an antenna whose electrical properties repeat periodically with algorithm of the logarithm of frequency. All dimensions increases in proportion to the distance from the origin. It has a number of dipoles of different length and spacing and all are fed with a balanced two wire line. The length and spacing are graduated in such a way that certain dimensions of adjacent elements bear a constant ratio to each other.

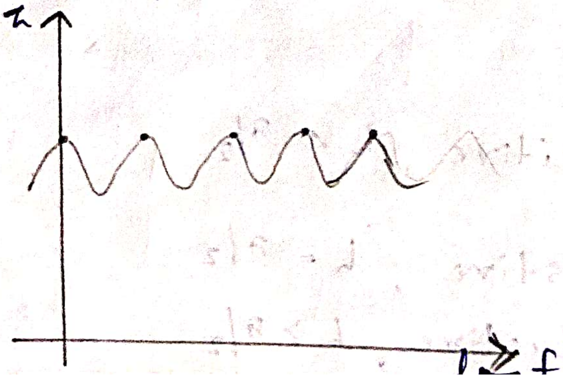
The dipole length increases such a way that the included angle α becomes a constant. The scale factor or design ratio is designated by τ whose value is less than 1.

<u>Design Ratio (τ):</u> $\frac{R_1}{R_2} = \frac{R_3}{R_4} = \cdots = \frac{R_n}{R_{n+1}} = \tau = \frac{L_2}{L_3} = \cdots = \frac{L_n}{L_{n+1}}$

$$\frac{R_n}{R_{n+1}} = \tau = \frac{L_n}{L_{n+1}}$$

$$\frac{S_{n+1}}{S_n} = \frac{L_{n+1}}{L_n} = \frac{1}{\tau} = k$$

The typical value of $\alpha = 30^\circ$ and $\tau = 0.7$



An LPDA has four different region of operation

1. Included transmission line region
2. loaded transmission line region

} Inactive region ($L < \lambda/2$)

3. Active region ($L \simeq \lambda/2$)
4. Reflective region ($L > \lambda/2$)

Inactive region:

* Antenna elements are short
* Elements present high capacitive impedance
* Element current is small and leads voltage by 90°
* Small radiation towards the apex.

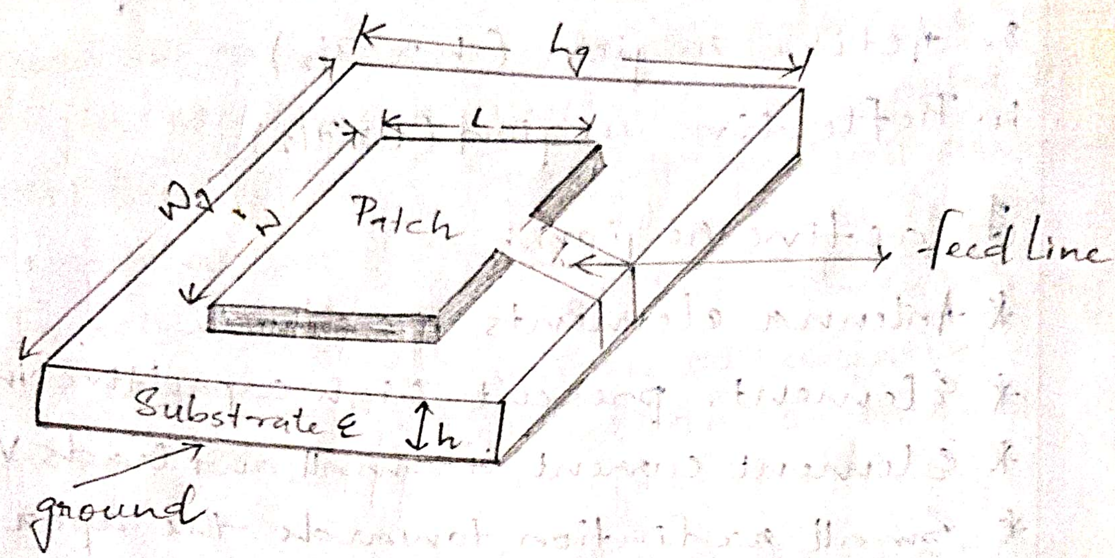
Active region:

* Dipole length approximately equal to $\lambda/2$
* Impedance offered is resistive.
* Element currents are large & inphase with voltage
* It presents large radiation towards apex

Reflective region:

* Dipole length are larger than resonant wavelength
* Impedance become inductive and current lags voltage

16/8/24 Microstrip patch Antenna Design.



Microstrip antennas are popular for low profile applications at frequencies above 100MHz. They usually consist of a rectangular patch on a dielectric coated ground plane. The patch and ground line has seperated by substrate and the radiating elements and feed lines are normally photo-etch on ~~the~~ dielectric substrate. The radiating patch may be square, rectangular or circular. Linear or circular polarization are achieved with microstrip antenna and arrays of RMPA [Rectangular Microstrip Patch Antenna] can be utilised for greater directivity. As the thickness of microstrip is very small, the waves generated in the dielectric substrate undergo reflection to the extend when they arrive the edges of strip resulting in radiation of only a small fraction of incident energy. Therefore, the antenna is considered to be very inefficient and it behaves more like a cavity rather than a radiator.

Limitation of RMPA:

* Narrow Bandwidth
* Low Gain
* High VSWR

Antenna Design:

The antenna parameters can be calculated by knowing the resonant frequency and dielectric constant ε_r and height h of the substrate. Resonant frequency and width of patch can be related using the equation,

$$\boxed{W = \frac{C_0}{2 f_r}\sqrt{\frac{2}{\varepsilon_r+1}}}$$

ε_r = Dielectric constant of Substrate

C_0 = Speed of light $= 3 \times 10^8\, m/s$

Effective diectric constant, ε_{eff}

$$\boxed{\varepsilon_{eff} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left[1+12h/w\right]^{-1/2}}$$

Due to fringing effect, electrically the size of antenna is increased by an amount of ΔL

$$\boxed{\frac{\Delta L}{h} = \frac{0.412\,(\varepsilon_{eff}+0.3)\left(\frac{w}{h}+0.264\right)}{(\varepsilon_{eff}-0.258)\left(\frac{w}{h}+0.8\right)}}$$

By knowing ΔL, we can calculate the length of patch,

$$\boxed{L = \frac{c_0}{2f_r\sqrt{\varepsilon_{eff}}} - 2\Delta L}$$

Length of ground / Length of Substrate

$$\boxed{L_g = 6h + L}$$

Width of ground, $\boxed{W_g = 6h + W}$

Feeding Mechanisms:

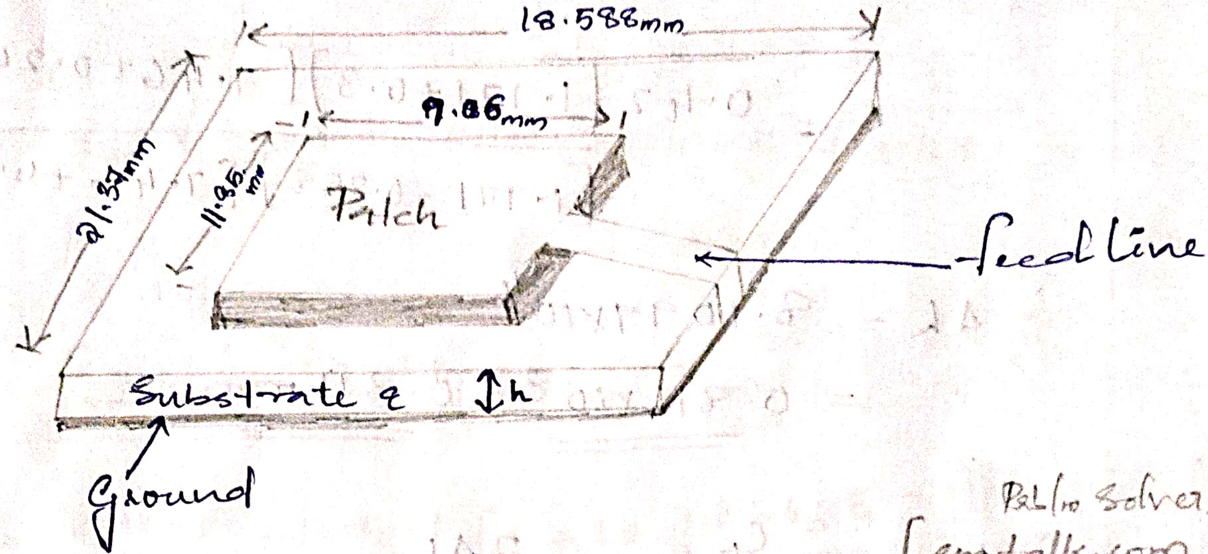
* Micro strip line feed
* Coaxial feed
* Aperture coupling feed
* Proximity feed.

6/24

Q. Design an RMPA using a substrate with dielectric constant 2.2, h = 0.1588 cm so as to resonate at 10 GHz. [2024]

Sol: $\epsilon_r = 2.2$, $h = 0.1588\,cm = 1.588\,mm$

$f_r = 10\,GHz$



Pablo Solver [emtalk.com]

$$W = \frac{c_0}{2f_r}\sqrt{\frac{2}{\epsilon_r+1}}$$

$$= \frac{3\times10^8}{2\times10\times10^9}\sqrt{\frac{2}{2.2+1}}$$

$$= 11.85\times10^{-3} = 11.85\,mm$$

$$\epsilon_{eff} = \frac{\epsilon_r+1}{2} + \frac{\epsilon_r-1}{2}\left[1+12h/w\right]^{-1/2}$$

$$= \frac{2.2+1}{2} + \frac{2.2-1}{2}\left[1+\frac{12\times1.588\times10^{-3}}{11.85\times10^{-3}}\right]^{-1/2}$$

$$= 1.971$$

$$\frac{\Delta L}{h} = 0.42 \frac{(\varepsilon_{eff}+0.3)\left(\frac{w}{h}+0.2644\right)}{(\varepsilon_{eff}-0.258)\left(\frac{w}{h}+0.8\right)}$$

$$\frac{\Delta L}{1.588\times10^{-3}} = 0.42 \frac{[1.971+0.3]\left[\frac{11.85\times10^{-3}}{1.588\times10^{-3}}+0.2644\right]}{[1.971-0.258]\left[\frac{11.85\times10^{-3}}{1.588\times10^{-3}}+0.8\right]}$$

$$= 0.42 \frac{[1.971+0.3][7.46+0.2644]}{[1.971-0.258][7.46+0.8]}$$

$$\therefore \Delta L = 8.1099\times10^{-4}$$

$$= \underline{\underline{0.811\times10^{-3}}} \quad \underline{\underline{0.811\,mm}}$$

$$\therefore L = \frac{C_0}{2f_r\sqrt{\varepsilon_{eff}}} - 2\Delta L$$

$$= \frac{3\times10^{8}}{2\times10\times10^{9}\sqrt{1.971}} - 2\times0.811\times10^{-3}$$

$$= \underline{\underline{9.06\times10^{-3}}} = \underline{\underline{9.06mm}}$$

$$L_g = 6h+L$$

$$= \underline{\underline{18.588\times10^{-3}}} = \underline{\underline{18.588\,mm}}$$

$$W_g = 6h+W$$

$$= 21.37\times10^{-3}$$

$$= \underline{\underline{21.37\,mm}}.$$

Q. Design a RMPA for wifi application [5.245 GHz] using for FR4 Substrate having height 1.6mm

ε_r for FR4 = 4.4

h = 1.6 mm

f_r = 5.245 GHz.

0.608540617

$$W = \frac{C_0}{2f_r}\sqrt{\frac{2}{\varepsilon_r+1}} = \frac{3\times10^8}{2\times5.245\times10^9}\sqrt{\frac{2}{4.4+1}}$$

$$= \underline{\underline{17.404mm}}$$

$$\varepsilon_{eff} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left[1+12h/w\right]^{-1/2}$$

$$= \frac{4.4+1}{2} + \frac{4.4-1}{2}\left[1+12\frac{1.6\times10^{-3}}{17.4\times10^{-3}}\right]^{-1/2}$$

$$= \underline{\underline{3.87}}.$$

$$\frac{\Delta L}{h} = 0.42\frac{[\varepsilon_{eff}+0.3]\left[\frac{w}{h}+0.2644\right]}{[\varepsilon_{eff}-0.258]\left[\frac{w}{h}+0.8\right]}$$

$$\frac{w}{h} = \frac{17.4\times10^{-3}}{1.6\times10^{-3}} = \underline{\underline{10.875}}$$

$$\frac{\Delta L}{1.6\times10^{-3}} = 0.42\frac{[3.87+0.3][10.875+0.2644]}{[3.87-0.258][10.875+0.8]}$$

$$\Delta L = \underline{\underline{0.726\times10^{-3}}}$$

$$\therefore \Delta L = \underline{\underline{0.726mm}}$$

$$L = \frac{C_0}{2f_r\sqrt{\varepsilon_{eff}}} - 2\Delta L$$

$$= \frac{3\times10^{8}}{2\times5.245\times10^{9}\times\sqrt{3.87}} - 2\times0.726\times10^{-3}$$

$$= \underline{\underline{13.08\,mm}}$$

$$L_g = 6h + L$$

$$= (6\times1.6\times10^{-3}) + (13.08\times10^{-3})$$

$$= 22.68\times10^{-3}$$

$$= \underline{\underline{22.68\,mm}}$$

$$W_g = 6h + W$$

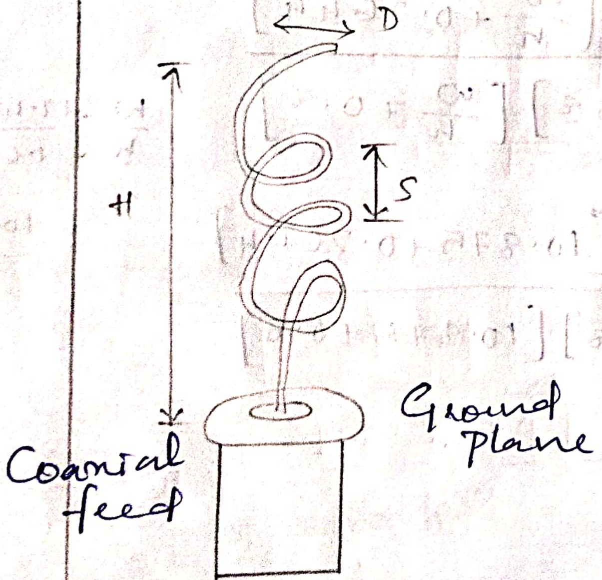
$$= (6\times1.6\times10^{-3}) + (17.40\times10^{-3})$$

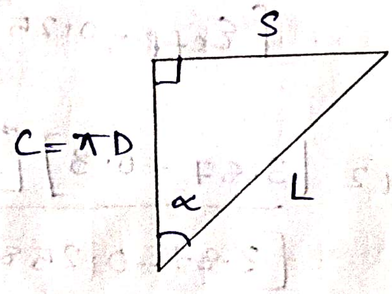
$$= 27\times10^{-3}$$

$$= \underline{\underline{27\,mm}}$$

22/8/24

Helix Antenna [Helical Antenna]





$$\boxed{\alpha = \tan^{-1}\frac{S}{\pi D} \qquad L = \sqrt{S^2 + C^2}}$$

It is a broadband VHF-UHF antenna which gives circular polarized waves. It consist of a thick copper wire wound in the shape of a screw thread and used as an antenna in conjunction with a metal plate called ground plate.

The helix is fed by coaxial cable and one end of the helix is connected to the center conductor and the other conductor is connected to the ground plane. The parameter on which mode of operation depends are

* diameter of helix (D)
* Term Spacing (s)
* Pitch angle (α) etc.

C ⇒ Circumference of helix

S ⇒ Spacing between the turns.

D ⇒ Diameter

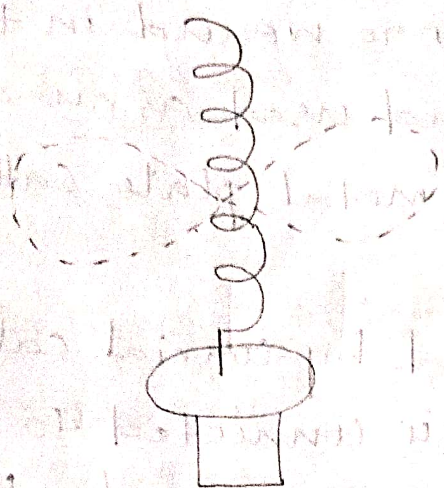
L ⇒ Length of one turn

α → Pitch angle.

The prominent mode of radiation are,

1. Normal / perpendicular mode of operation.
2. Axial mode / End fire mode.

Normal mode of operation.



Normal mode of radiation is obtained when the dimensions of antenna are very small compared to operating wavelength λ. Here the radiation is maximum in the direction perpendicular to helix axis. Here the the radiation fields are similar to the radiations of loops and short dipoles.

i.e., helix is equivalent to the series combination of short dipole and loop.

The field of a small group is given by

$$E_\phi = \frac{j120\pi^2 [I] \sin\theta}{r} \cdot \frac{A}{\lambda^2} \qquad A = \frac{\pi D^2}{4}$$

The radiation generated by the short dipole,

$$E_\theta = j \frac{60\pi [I] \sin\theta}{r} \frac{S}{\lambda}$$

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$$AR = \left|\frac{E_\theta}{E_\phi}\right| = \frac{\dfrac{j\,60\pi[I]\sin\theta}{r}\dfrac{S}{\lambda}}{\dfrac{j\,120\pi^2[I]\sin\theta}{r}\times\dfrac{A}{\lambda^2}}$$

AR ↓ Axial Ratio

$$= \frac{1}{2\pi}\,\frac{S}{\lambda}\times\frac{\lambda^2}{A}$$

Here A = Area of helix.

$$AR = \frac{S\lambda}{2\pi\times\frac{\pi D^2}{4}} = \frac{2S\lambda}{\pi^2 D^2} = \frac{2S\lambda}{C^2}$$

As it supports circular polarization, AR = 1

$$\therefore\ 1 = \frac{2S\lambda}{C^2}$$

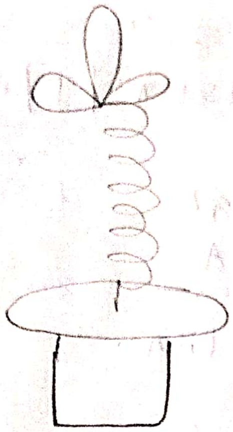
$$S = \frac{C^2}{2\lambda} = \frac{(\pi D)^2}{2\lambda}.$$

If $E_\theta = 0$, axial ratio becomes zero, we get horizontal polarizing signal.

If $E_\phi = 0$, then axial ratio becomes infinity and we will get vertical polarization.

If $\frac{E_\theta}{E_\phi} = 1$, axial ratio becomes oval and we get circular polarization.

Axial Mode of Operation.



Here the maximum radiation will be in the direction of helix axis, here the dimension of helix are comparable to λ.

$AR = 1 + \frac{1}{2N}$ $\quad$ N = Number of turns.

The expression for terminal impedence, r

$r = 140 \frac{C}{\lambda}$.

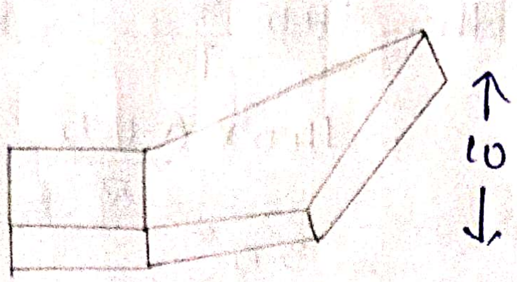
Q. Calculate the power gain of an optimum horn antenna with a square operator of 10λ on one side.

Sol: Here the power gain $G_p = \frac{4.5 A}{\lambda^2}$

Where A is the area, $10\lambda \times 10\lambda$

$A = 100\lambda^2$

$\therefore G_p = \frac{4.5 \times 100 \cancel{\lambda^2}}{\lambda^2} = \underline{\underline{450}}$



$10 \log 450 =$ power gain (in dB)

Power Gain $= 26.5321$ dB

Q. Find the power gain of a parabolised reflector of open mouth aperture of 10λ.

Sol: $G_p = 6 \times \left(\frac{D}{\lambda}\right)^2$ D is the diameter of aperture in λ.

$6 \times \left(\frac{10\lambda}{\lambda}\right)^2 = \underline{600}$

Power Gain $P_g = 10 \log 600$

$= 27.7815$ dB

Q. Find the beam width with the first nulls and power gain of a 2m ~~600~~ parabolised reflector operation at 600 MHz.

Sol: $f = \frac{c}{\lambda}$ $c = 3 \times 10^8$

$f = 600 \times 10^6$

$G_p = 6 \left(\frac{D}{\lambda}\right)^2$

$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{6 \times 10^6} = \underline{0.05}$

$G_p = 6 \times \left(\frac{2}{0.05}\right)^2 = 9600$

$G_p = \underline{39.8227 \text{ dB}}$.

first null beam width $= 140 \frac{\lambda}{D}$
(FNB)

$= 140 \times \frac{0.05}{2}$

$= \underline{\underline{3.5^\circ}}$

Q. Estimate the diameter of parabolised reflector antenna required to produce FNB of 10° at 3GHz.

Sol: $f = \frac{c}{\lambda}$

$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^9} = \frac{1}{10} = \underline{\underline{0.1}}$

FNB $= 140 \left(\frac{\lambda}{D}\right)$

$= \frac{140 \times 0.1}{D}$

$\therefore D = \frac{140 \times 0.1}{10} = \underline{\underline{1.4 m.}}$

Q. A parabolic reflector required to half a power gain of 1000 at 3GHz. find the mouth diameter and Beam width of antenna.

Sol: $G_p = 6 \times \left(\frac{D}{\lambda}\right)^2$

$1000 = 6 \left(\frac{D}{\lambda}\right)^2$

$\frac{1000}{6} = \frac{D^2}{\lambda^2}$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^7} = \underline{\underline{0.1}}$$

$$D^2 = \frac{10}{6} = \underline{\underline{1.6666}}$$

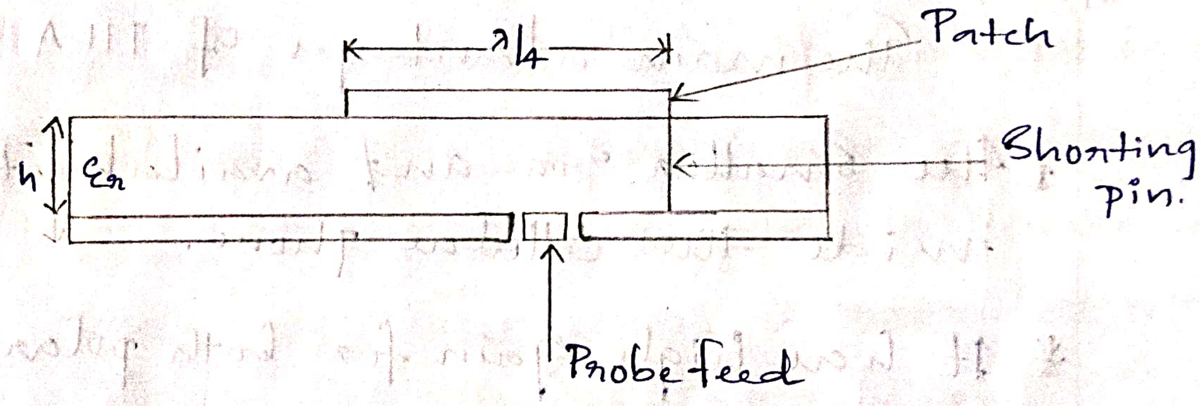
$$\therefore D = \underline{\underline{1.290\,m}}.$$

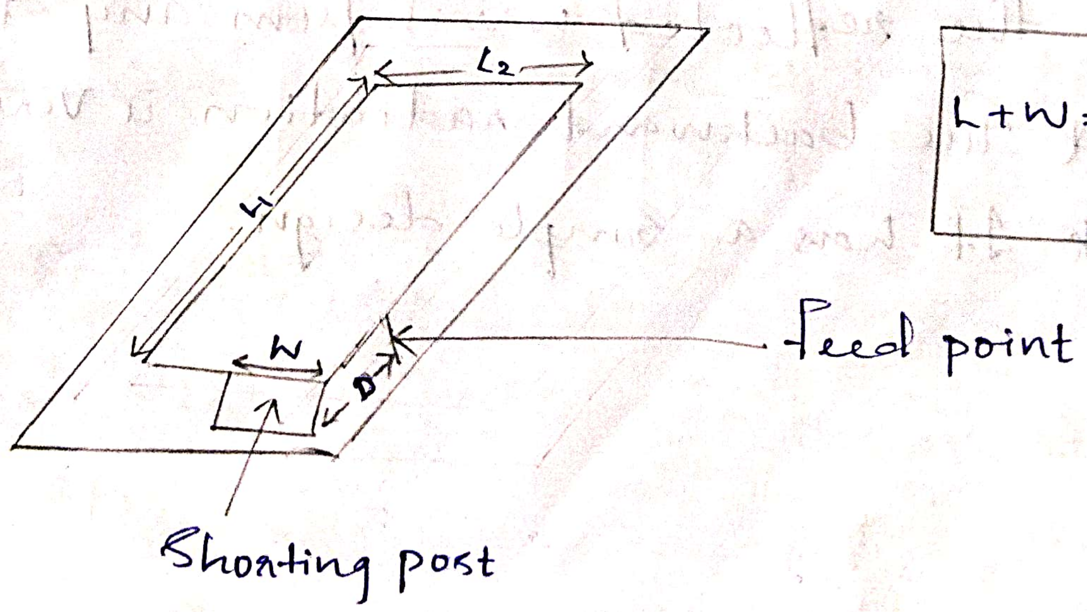
$$FNBW = \frac{140 \times 0.1}{1.2}$$

27/8/24

Antennas for Mobile Stations and Base Station.

Planar Inverted F antenna [PIFA]





$$\boxed{L + W = \frac{\lambda}{4\sqrt{\varepsilon_r}}}$$

It is a quarter wavelength antenna with a good SAR [Specific Absorption Rate] properties. The antenna resembles an inverted F and is popular because it has low profile and omni directional pattern. It is a quarter wave resonant antenna with a shorting pin at one end. The feed is placed between the open end and shorted end and its position controls the impedance. The closer the ~~field~~ feed to the shorting pin, lesser will be the impedance & the impedance can be increased by moving it (feed) farther from the shorting edge.

The main advantages of PIFA are:

* The smaller size and availability to insert inside the cellular phone.

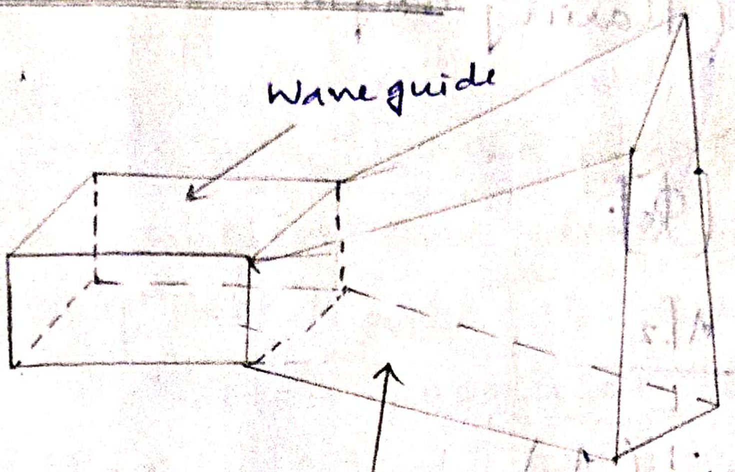
* It has high gain for both polarisation states vertically & horizontally and can receive the reflected waves from any directions.

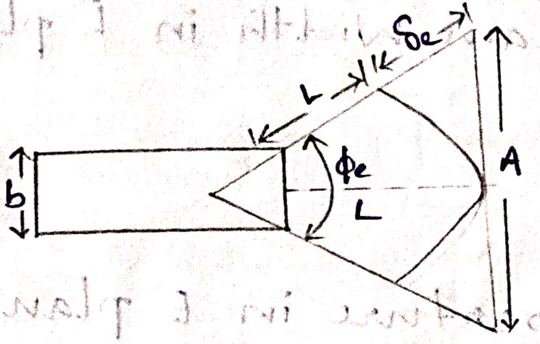
* The backward radiation is very less.

* It has a simple design.

1/8/24

Horn Antenna





A horn antenna may be considered as a flared out wave guide by which the directivity is ~~increased~~ improved and the defraction is reduced. The operating frequency of Horn antenna is 300MHz to 30GHz. The horn is provided with an extended aperture to make abrupt discontinuity of wave guide into a gradual transformation. From figure, by using pythagorous relation,

$$(L+\delta e)^2 = (A/2)^2 + L^2$$

$$\cancel{L^2} + \delta e^2 + 2\delta e L = \frac{A^2}{4} + \cancel{L^2} \qquad (\text{neglect } \delta e^2)$$

$$2\delta e L = \frac{A^2}{4}$$

$$\boxed{L = \frac{A^2}{8\delta_e}}$$ (flaring length)

Flaring angle $[\phi_e]$.

$$\tan \phi_e/2 = \frac{A/2}{L}$$

$$\boxed{\phi_e = 2\tan^{-1}\left(\frac{A}{2L}\right)}$$

The half-power beam width in E plane,

$$\boxed{\theta_E = \frac{56\lambda}{d_E}}$$

where d_E is the aperture in E plane

The half-power beam width in H plane,

$$\boxed{\theta_H = \frac{67\lambda}{d_H}}$$ d_H = aperture in H plane

Directivity, $\boxed{D = \frac{7.5 A_p}{\lambda^2}}$ A_p = physical aperture $= d_E \cdot d_H$

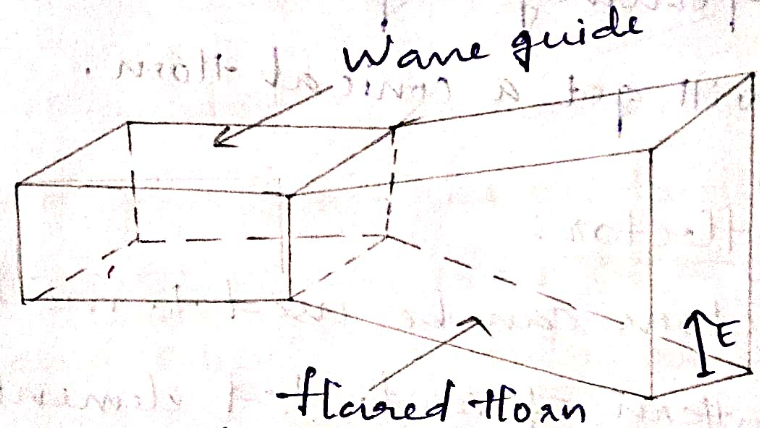
Power Gain $\boxed{G = \frac{4.5 A_p}{\lambda^2}}$

Types of Horn Antenna.

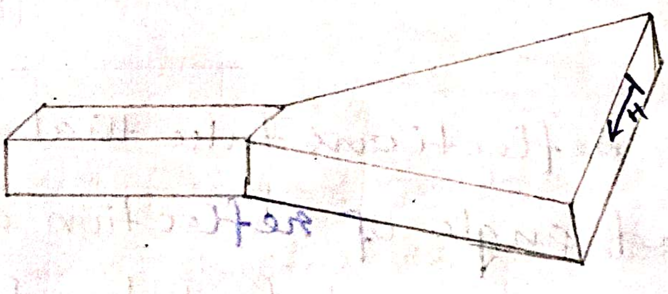
1. **Sectoral Horn:** In this type, the flaring is only in one direction.

   * The flaring in the direction of electric Vector produces E-plane Horn.

   * The flaring in the direction of magnetic Vector produces H-plane Horn antenna.

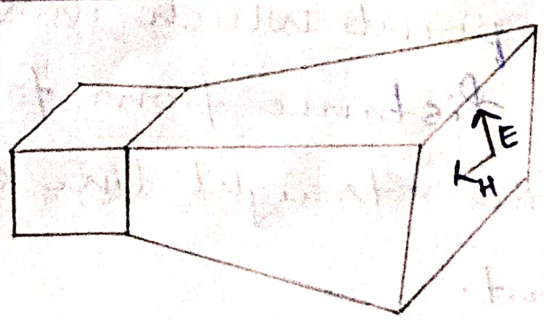


E-plane Horn.



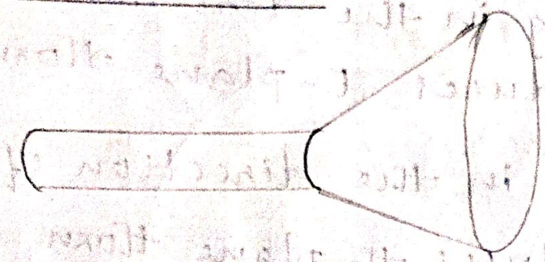
H-plane Horn.

2. **Pyramidal Horn:**



In pyramidal horn flaring introduced to both plane i.e., both E and H walls of rectangular wave guide.

3. Conical Horn:

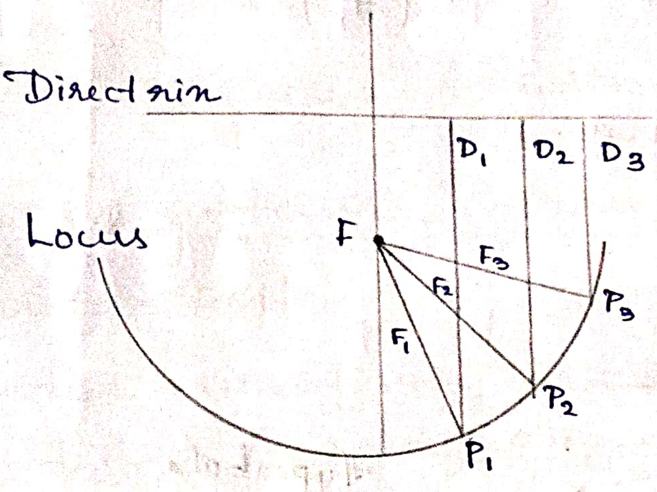


If the flaring is given to a circular wave guide, we will get a conical Horn.

21/8/24 Parabolic Reflector:

The reflectors can be used to modify the radiation pattern of radiated element. Reflected antenna can offer much gains than horn antenna. The frequency range used for the application of parabolic reflector is 300 MHz to 300 GHz.

The law of reflections take that the angle of incidence and angle of reflection are equal. This law along with a parabola helps for beam focus. The locus of a point which moves in such a way that the distance from focus and the distance from the straight line called directrix is a constant.



A parabolic reflector has two parts
1. Feed antenna
2. Parabolic reflector

When it is used as a transmitter the signals from the feed antenna focus on to the parabola. The wave now gets reflected as a collimated wave front and when it is used as a receiver all the electromagnetic waves which hits the parabola gets reflected onto the feed antenna.

Design Equation:

1. Gain: $10 \log k\left(\frac{\pi D}{\lambda}\right)^2$

$k \Rightarrow$ Efficiency factor
$D \Rightarrow$ Diameter of parabola.

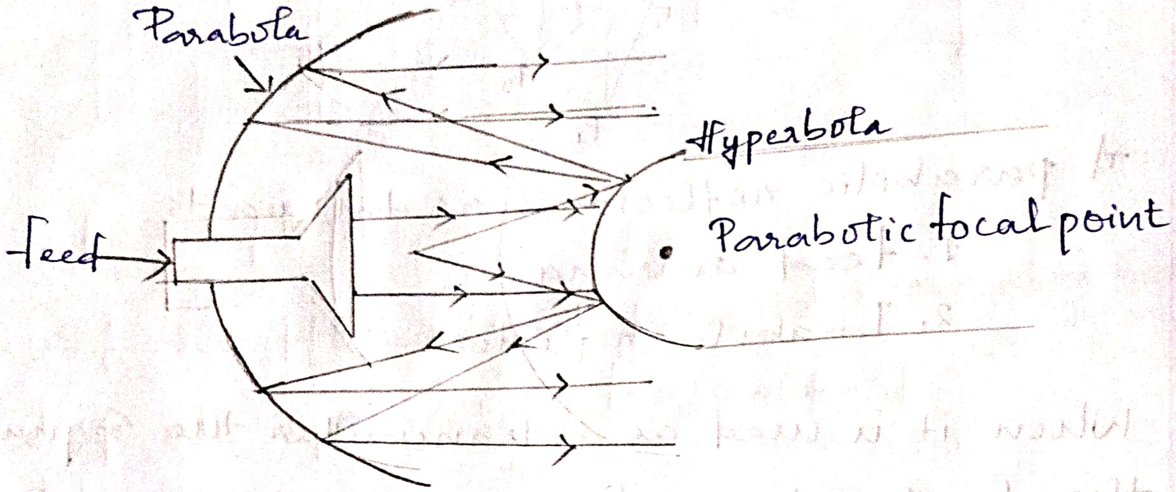
2. Half Power Beam Width, $HPBW = \frac{70\lambda}{D}$

3. First Narrow Beam Width, $FNBW = \frac{140\lambda}{D}$

4. Directivity, $D = 6.4\left(\frac{D}{\lambda}\right)^2$

5. Power gain, $G_p = 9.87 \left(\frac{D}{\lambda}\right)^2$

Cassegrain Antenna:



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Module-3
ANTENNA ARRAYS

An antenna array is a system of similar antennas oriented similarly to get greater directivity in the desired direction. The total field produced by an antenna array at a far distant point is the vector sum of fields produced by the individual antennas of the array system.

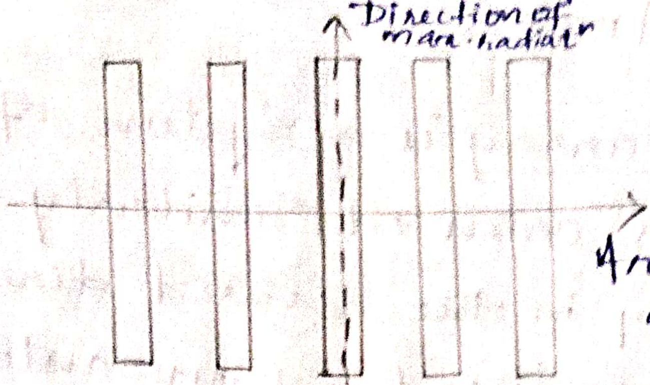
An array is said to be linear if the individual antennas of that array are equally spaced. The individual antennas of array are termed as elements.

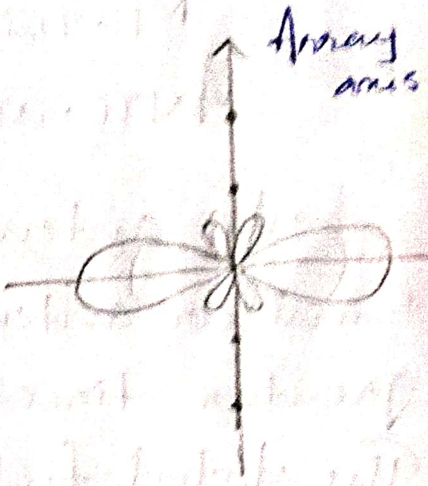
A uniform linear array is the one in which the elements are fed with currents of equal magnitude with a progressive phase shift.

Various types/Forms of Antenna.

* Broad Side Array
* End-fire Array
* Co-linear Array
* Parasite Array.

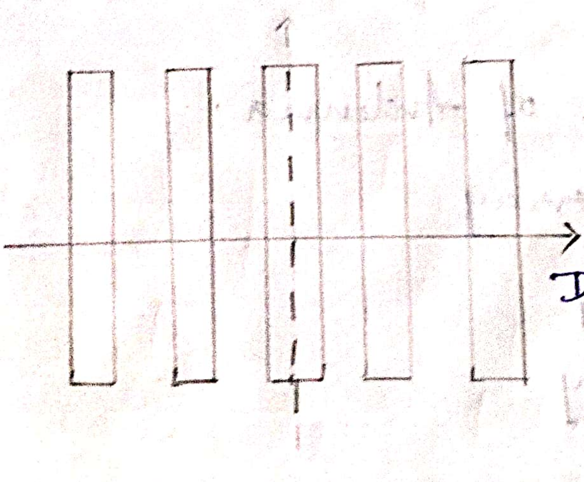
Broad Side Array. [BSA]





Broad Side array is the one in which a number of identical parallel antennas are Setup along a line drawn perpendicular to its axis. Here the antenna elements are equally Spaced and each element is fed with currents of equal magnitude and inphase and fires maximum radiation in the direction perpendicular to the line of array axis (broad-Side)

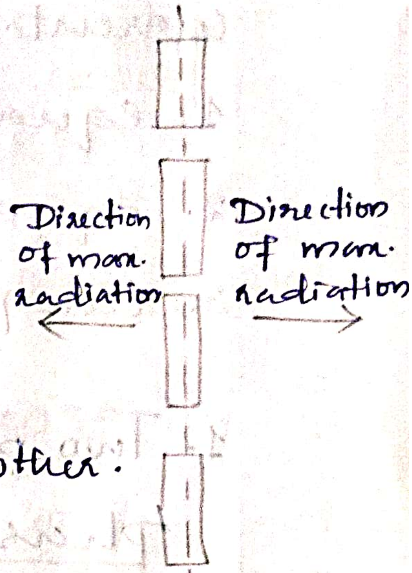
End-Fire Array.

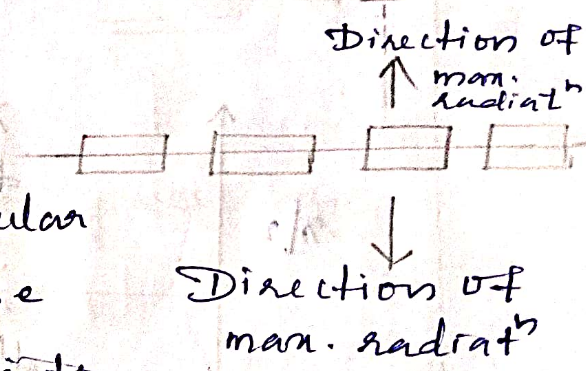


Here the antenna arrangement is same as that of BSA but are fed with currents of equal magnitude whose phase changes progressively along the line to make the pattern unidirectional.

Co-linear Arrays.

It is a category of antenna array in which the elements are arranged co-anially by mounting the elements end-to-end in straight line or stacking them one over the another. The individual antennas are fed with currents as in broad side antenna. The radiation pattern has a cirular symetry with its main lobe at all points perpendicular to principle anis.





Parasitic Arrays.

In this type of arrays, the elements are fed parasitically to reduce feed problem. The parasitic not ~~drives~~ fed directly instead it is derived power from radiation of nearby driven element.

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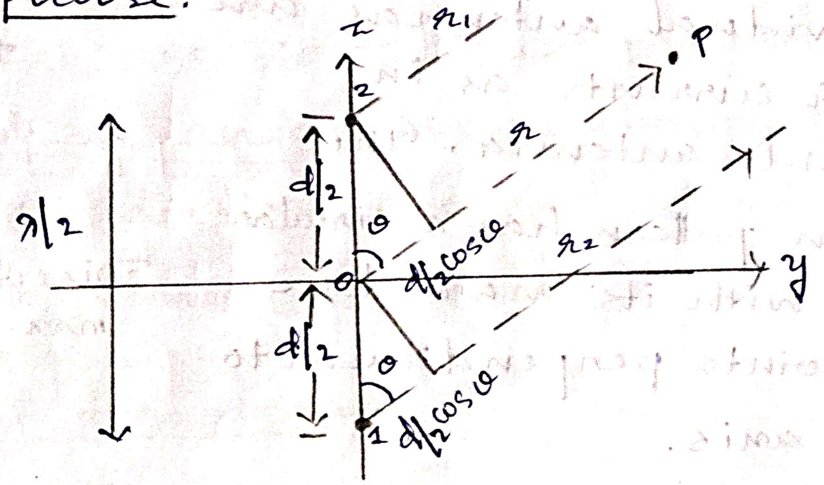
Array of point Sources.

The directivity of point source can be increased by increasing the number of point sources together with various parameters such as phase and amplitude relationships with current. These parameters can be

1. equal amplitude and phase.
2. equal amplitude and opposite phase.
3. unequal amplitude and opposite phase.

1. Two point Sources with Same amplitude and Phase:



$$\frac{d}{2} + \frac{d}{2} = \frac{\lambda}{2}$$

Consider two point sources placed symmetrically w.r.t origin of z axis and are excited with currents of same amplitude and phase. Let P be a far distant point where the total field is to be calculated.

Let E_0 be the strength of field at point P due to either of the point sources, then the total field at P can be calculated by taking vector sum of fields and considering the amount of phase difference due to path difference.

The path difference $= \frac{d}{2}\cos\theta + \frac{d}{2}\cos\theta$

$= d\cos\theta.$

Phase difference $(\psi) = \frac{2\pi}{\lambda} \times$ path difference

$= \frac{2\pi}{\lambda} \times d\cos\theta$

$= \beta d\cos\theta$

where $\left\{\beta = \frac{2\pi}{\lambda}\right\}$

The total field at point P is given by,

$$E = E_0 e^{-j\psi/2} + E_0 e^{j\psi/2}$$

$$= E_0 \left[e^{-j\psi/2} + e^{j\psi/2}\right]$$

$$= 2E_0 \left[\frac{e^{-j\psi/2} + e^{j\psi/2}}{2}\right]$$

$$= 2E_0 \cos\psi/2$$

$\left\{\frac{e^{j\theta} + e^{-j\theta}}{2} = \cos\theta\right.$

Normalised field $E_n = \frac{E}{2E_0} = \frac{2E_0\cos\psi/2}{2E_0}$

$$E_n = \cos\psi/2$$

Substituting for ψ,

$$= \cos\left(\frac{\beta d \cos\theta}{2}\right) \qquad \beta d = \frac{2\pi}{\lambda} \times \frac{\lambda}{2}$$

$$= \pi$$

$$E_n = \cos(\pi/2 \cos\theta)$$

Case 1: Direction of maximum radiation.

$[E_n = 1]$

$$\cos(\pi/2 \cos\theta) = \pm 1$$

$$\pi/2 \cos\theta = 0$$

$$\cos\theta = 0$$

$$\therefore \theta = 90^\circ, 270^\circ$$

Case 2: Direction of minimum radiation/null

$[E_n = 0]$

$$\cos(\pi/2 \cos\theta) = 0$$

$$\cancel{\pi/2} \cos\theta = \pm \cancel{\pi/2}$$

$$\cos\theta = \pm 1$$

$$\therefore \theta = 0^\circ, 180^\circ$$

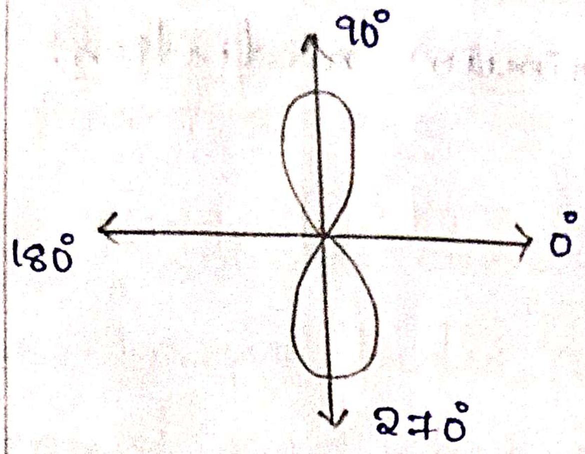
Case 3: Direction of Half power points.

$\left[E_n = \frac{1}{\sqrt{2}}\right]$ $\cos(\pi/2 \cos\theta) = \pm \frac{1}{\sqrt{2}}$

$$\frac{\pi}{2} \cos\theta = \pm \frac{\pi}{4}$$

$$\cos\theta = \pm 1/2$$

$$\therefore \theta = 60^\circ, 120^\circ$$



2. Two point Sources with Same amplitude and ~~different~~ opposite phase.

Here Source 1 is in opposite phase with Source 2. Therefore the Superposition of field is given by,

$$E = E_0 e^{+j\psi/2} - E_0 e^{-j\psi/2} \qquad \left\{ \frac{e^{j\theta} - e^{-j\theta}}{2j} = \sin\theta \right.$$

$$= E_0 \left[e^{j\psi/2} - e^{-j\psi/2} \right] = 2jE_0 \left(\frac{e^{j\psi/2} - e^{-j\psi/2}}{2j} \right)$$

$$E = 2jE_0 \sin(\psi/2)$$

Normalised field $E_n = \frac{E}{2E_0} = j\sin(\psi/2)$

Here j represents $90°$ phase shift.

$$E_n = \sin\left(\frac{\beta d\cos\theta}{2}\right) = \sin\left(\frac{\frac{2\pi}{\cancel{\lambda}} \frac{\cancel{\lambda}}{2}\cos\theta}{2}\right)$$

$$= \sin(\pi/2 \cos\theta)$$

Case 1. Direction of maximum radiation:

$[E_n = 1]$

$$\sin(\pi/2 \cos\theta) = \pm 1$$

$$\pi/2 \cos\theta = \pi/2$$

$$\cos\theta = \pm 1$$

$$\therefore \theta = 0^\circ, 180^\circ$$

Case 2: Direction of Null

$[E_n = 0]$

$$\sin(\pi/2 \cos\theta) = 0$$

$$\pi/2 \cos\theta = 0$$

$$\cos\theta = 0$$

$$\theta = 90^\circ, 270^\circ$$

Case 3: Direction of half power radiation.

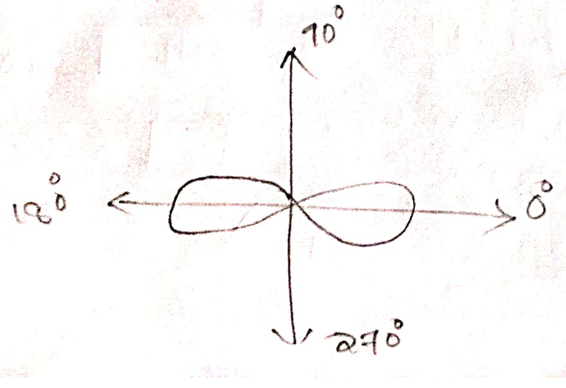
$[E_n = 1/\sqrt{2}]$

$$\sin(\pi/2 \cos\theta) = 1/\sqrt{2}$$

$$\pi/2 \cos\theta = \pi/4$$

$$\cos\theta = \pm 1/2$$

$$\therefore \theta = 60^\circ, 120^\circ$$



3. Two point Sources with unequal amplitude and phase.

Let us consider a condition in which the amplitudes of two point Sources are not equal and have a phase difference delta (δ). Let E_1 and E_2 be the amplitude of field at P. Let δ be the phase angle by which the current at Source 2 leads the currents at Source 1. Then the phase difference b/w the two Sources is given by,

$$\psi = \beta d \cos\theta + \delta \qquad \cancel{\psi = \beta d \cos\theta + \delta}.$$

The resultant field can be,

$$E = E_1 e^{-j\psi/2} + E_2 e^{j\psi/2}$$

Further for Simplicity, let the System be displaced so that Source 1 coincides with Origin, then the above equation becomes

$$E = E_1 e^{j0} + E_2 e^{j\psi}$$

$$= E_1 \left[1 + \frac{E_2}{E_1} e^{j\psi} \right] = E_1 \left[1 + k e^{j\psi} \right]$$

$$= E_1 \left[1 + k \left[\cos\psi + j \sin\psi \right] \right]$$

$$= E_1 \left[1 + k\cos\psi + jk\sin\psi \right]$$

∴ The magnitude

$$\boxed{|E| = E_1\sqrt{(1+k\cos\psi)^2+(k\sin\psi)^2}}$$

Phase,

$$\boxed{\angle E = \tan^{-1}\left[\frac{k\sin\psi}{1+k\cos\psi}\right]}$$

✓ <u>Principle of pattern multiplication.</u>

A non-isotropic source is that source which send more or less radiation in a particular direction. Hertz dipole is an example for non-isotropic source.

It has already derived that the field at a given far distant point due to two isotropic sources having phase difference δ as,

$$E = 2E_0\cos\psi/2 \quad \text{where} \quad \psi = \beta d\cos\theta + \delta$$

For Hertizian dipole, the field varies with angle θ ie; $E = E_0\sin\theta$

and the resultant field for the case of array of non-isotropic point sources becomes,

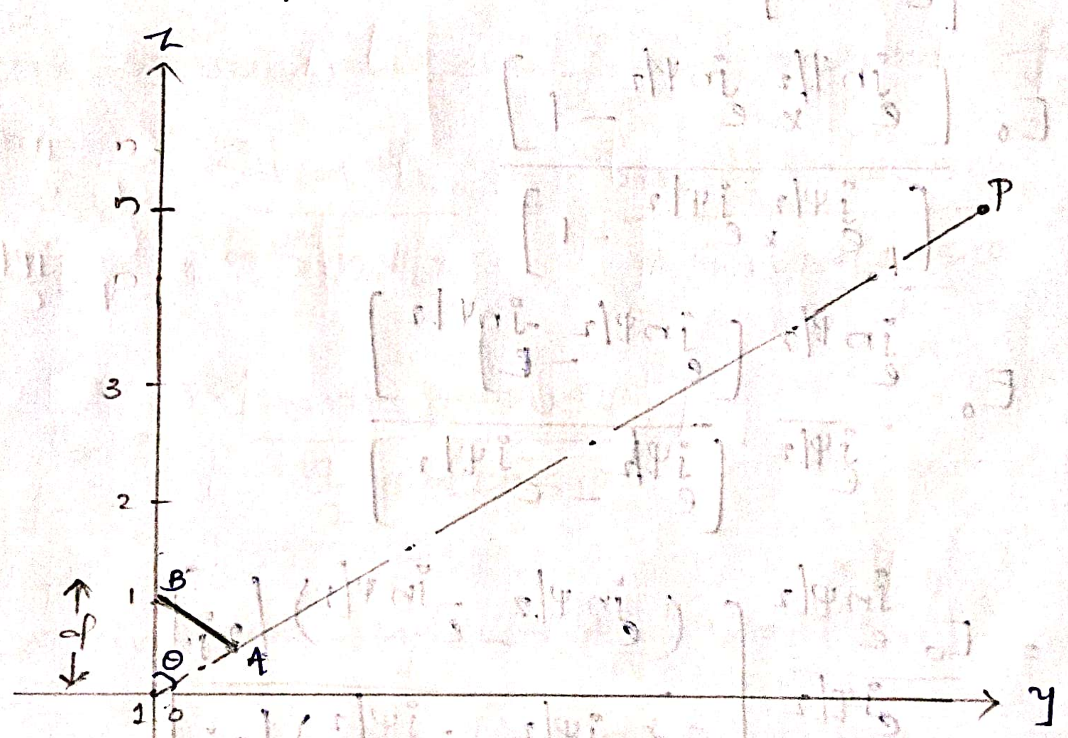
$$E = 2E_0\cos\psi/2 \times \sin\theta$$

The normalised pattern,

$$E_n = \frac{E}{2E_0} = \underbrace{\cos \psi/2}_{\text{Isotropic pattern}} \times \underbrace{\sin \theta}_{\text{non-isotropic pattern.}}$$

ie; E_n = pattern of individual isotropic point source multiplied by pattern of non-isotropic point source.

Array of n-isotropic point source with Equal amplitude and phase:



$$E = E_0 e^{j0\psi} + E_0 e^{j\psi} + E_0 e^{j2\psi} + \cdots + E_0 e^{j(n-1)\psi}$$

Here, n isotropic sources are placed on z axis and the total field generated at P can be expressed as,

$$E = E_0 e^{j0\psi} + E_0 e^{j\psi} + E_0 e^{j2\psi} + \cdots + E_0 e^{j(n-1)\psi}$$

3 $E = E_0\left[1 + e^{j\psi} + e^{j2\psi} + \cdots + e^{j(n-1)\psi}\right]$ —— ①

Multiply both sides with $e^{j\psi}$

$$E e^{j\psi} = E_0\left[e^{j\psi} + e^{j2\psi} + \cdots + e^{jn\psi}\right] \quad \text{—— ②}$$

② − ①

$$E\left[e^{j\psi} - 1\right] = E_0\left[e^{jn\psi} - 1\right] \quad \text{—— ③}$$

$$\cancel{\frac{E}{E_0} = \frac{\left[e^{jn\psi} - 1\right]}{\left[e^{j\psi} - 1\right]}} \qquad E = E_0 \frac{\left[e^{jn\psi} - 1\right]}{\left[e^{j\psi} - 1\right]}$$

$$E = E_0 \frac{\left[e^{jn\psi/2} \times e^{jn\psi/2} - 1\right]}{\left[e^{j\psi/2} \times e^{j\psi/2} - 1\right]} \qquad \left\{ \begin{array}{l} 1 = e^{jn\psi/2} \times e^{-jn\psi/2} \\ 1 = e^{j\psi/2} \times e^{-j\psi/2} \end{array} \right\}$$

$$= E_0 \frac{e^{jn\psi/2}}{e^{j\psi/2}} \frac{\left[e^{jn\psi/2} - e^{-jn\psi/2}\right]}{\left[e^{j\psi/2} - e^{-j\psi/2}\right]}$$

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$$E = E_0 \frac{e^{jn\psi/2}}{e^{j\psi/2}} \left[\frac{\left(e^{jn\psi/2} - e^{-jn\psi/2}\right)/2j}{\left(e^{j\psi/2} - e^{-j\psi/2}\right)/2j}\right]$$

$$= E_0 e^{j(n-1)\psi/2} \left[\frac{\sin n\psi/2}{\sin \psi/2}\right]$$

$$= E_0 e^{j\phi} \left[\frac{\sin n\psi/2}{\sin \psi/2}\right]$$

Where, $\phi = (n-1)\psi/2$

If the array is placed symmetrically above the origin, then,

$$E = E_0 \left[\frac{\sin n\psi/2}{n \sin \psi/2} \right]$$

$$\boxed{\text{Array factor} = \frac{\sin n\psi/2}{n \sin \psi/2}}$$

Directivity of n-element Linear Array.

* **Broadside Array**

The normalised field array factor,

$$E_n = \frac{\sin n\psi/2}{n \sin \psi/2} = \frac{\sin \frac{n}{2}\beta d\cos\theta}{n \sin \frac{\beta d\cos\theta}{2}}$$

for very small θ,
$\sin\theta = \theta$

$$E_n = \frac{\sin n/2\, \beta d\cos\theta}{\frac{n}{2}\beta d\cos\theta}$$

Radiation Intensity, $U = |E_n|^2$

$$U = \left| \frac{\sin \frac{n}{2}\beta d\cos\theta}{\frac{n}{2}\beta d\cos\theta} \right|^2 \quad \text{—— (1)}$$

Directivity D, $= \dfrac{U_{max}}{U_{avg}}$

$$U_{avg} = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} U \sin\theta \, d\theta \, d\phi \quad \text{——} \quad (2)$$

Substitute (1) in eqn (2);

$$\int_0^{2\pi} d\phi = [\phi]_0^{2\pi} = 2\pi$$

$$U_{avg} = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\left|\frac{\sin \frac{n}{2}\beta d\cos\theta}{\frac{n}{2}\beta d\cos\theta}\right|^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{2\pi}{4\pi}\int_0^{\pi}\left|\frac{\sin \frac{n}{2}\beta d\cos\theta}{\frac{n}{2}\beta d\cos\theta}\right|^2 \sin\theta \, d\theta$$

$$U_{avg} = \frac{1}{2}\int_0^{\pi}\left|\frac{\sin \frac{n}{2}\beta d\cos\theta}{\frac{n}{2}\beta d\cos\theta}\right|^2 \sin\theta \, d\theta \quad \text{——} \quad (3)$$

Let $z = \frac{n}{2}\beta d\cos\theta$ when $\theta = 0,\ z = \frac{n\beta d}{2}$

$dz = -\frac{n}{2}\beta d \sin\theta \, d\theta$ $\quad \theta = \pi,\ z = -\frac{n\beta d}{2}$

$$\sin\theta \, d\theta = \frac{dz}{-\frac{n\beta d}{2}}$$

$\therefore$ (3) $\Rightarrow$

$$U_{avg} = \frac{1}{2}\int_{\frac{n\beta d}{2}}^{-\frac{n\beta d}{2}}\left|\frac{\sin z}{z}\right|^2 \frac{dz}{-\frac{n\beta d}{2}}$$

$$= -\frac{1}{n\beta d}\int_{\frac{n\beta d}{2}}^{-\frac{n\beta d}{2}}\left|\frac{\sin z}{z}\right|^2 dz$$

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$$U_{avg} = \frac{1}{n\beta d}\int_{-\frac{n\beta d}{2}}^{\frac{n\beta d}{2}} \left|\frac{\sin z}{z}\right|^2 dz \quad \text{———} \quad (4)$$

For a large element array, n tends to infinity, $\therefore \frac{n\beta d}{2} \to \infty$

$$(4) \Rightarrow U_{avg} = \frac{1}{n\beta d}\int_{-\infty}^{\infty} \left|\frac{\sin z}{z}\right|^2 dz$$

Since $\int_{-\infty}^{\infty} \left|\frac{\sin z}{z}\right|^2 dz = \pi$, we get

$$U_{avg} = \frac{\pi}{n\beta d}.$$

$\therefore$ Directivity $D = \frac{U_{max}}{U_{avg}}$, $U_{max} = 1$ at $\theta = 90°$

$$\therefore D = \frac{1}{\frac{\pi}{n\beta d}} = \frac{n\beta d}{\pi}$$

But we know $\beta = \frac{2\pi}{\lambda}$

$$= \frac{2n\pi d}{\lambda\pi} = \frac{2nd}{\lambda}$$

$$\boxed{D = \frac{2nd}{\lambda}}$$

Q. Calculate the directivity of a broad side array having element spacing 0.3λ and 10 elements. What is the length of array?

Sol: $D = \frac{2nd}{\lambda}$ $L = (n-1)d$

Given, $d = 0.3\lambda$
$n = 10$

$\therefore D = \frac{2 \times 10 \times 0.3 \times \cancel{\lambda}}{\cancel{\lambda}}$

$\underline{\underline{D = 6}}$

$L = (10-1)0.3\lambda$
$= 9 \times 0.3 \times \lambda$
$= \underline{\underline{2.7\lambda}}$

Q What is the directivity of end-fire array of 5 elements with a spacing of 0.2λ.

Sol: $D = \frac{4nd}{\lambda}$ $n = 5$
$d = 0.2\lambda$

$= \frac{4 \times 5 \times 0.2 \times \lambda}{\lambda}$

$D = \underline{\underline{4}}$

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* Directivity of End fire Array.

for end fire array,

$$\psi = \beta d \cos\theta + \delta \quad \text{—— (1)}$$

Take $\theta = 0$, $\psi = 0$

$$\therefore \quad 0 = \beta d \cos 0 + \delta$$

$$\delta = -\beta d \quad \text{—— (2)}$$

(2) in (1)

$$\psi = \beta d \cos\theta - \beta d$$

$$= \beta d [\cos\theta - 1]$$

$$E_n = \frac{\sin n\psi/2}{n \sin \psi/2}$$

Substituting the value of ψ,

$$E_n = \frac{\sin \frac{n}{2}\beta d(\cos\theta - 1)}{n \sin \frac{\beta d}{2}(\cos\theta - 1)}$$

for very small θ, $\sin\theta = \theta$.

$$= \frac{\sin \frac{n\beta d}{2}[\cos\theta - 1]}{\frac{n\beta d}{2}[\cos\theta - 1]}$$

$$U = |E_n|^2$$

$$= \left| \frac{\sin \frac{n\beta d}{2}[\cos\theta - 1]}{\frac{n\beta d}{2}[\cos\theta - 1]} \right|^2$$

$$U = \left| \frac{\sin z}{z} \right|^2 \quad \text{where } z = \frac{n\beta d[\cos\theta - 1]}{2}$$

Directivity $D = \frac{U_{max}}{U_{avg}}$

$$\int_0^{2\pi} d\phi = [\phi]_0^{2\pi} = 2\pi$$

$$U_{avg} = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} U \sin\theta \, d\theta \, d\phi$$

$$= \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} \left|\frac{\sin z}{z}\right|^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{2\pi}{4\pi}\int_0^{\pi} \left|\frac{\sin z}{z}\right|^2 \sin\theta \, d\theta$$

$$U_{avg} = \frac{1}{2}\int_0^{\pi} \left|\frac{\sin z}{z}\right|^2 \sin\theta \, d\theta \quad \text{--- (3)}$$

Since $z = \frac{n\beta d}{2}[\cos\theta - 1]$

$dz = -\frac{n\beta d}{2}\sin\theta \, d\theta$, $\sin\theta \, d\theta = \frac{dz}{-n\beta d/2}$

When $\theta = 0$, $z = 0$

$\theta = \pi$, $z = -n\beta d$

$$\therefore (3) \Rightarrow U_{avg} = \frac{1}{2}\int_0^{-n\beta d} \left|\frac{\sin z}{z}\right|^2 \frac{dz}{-n\beta d/2}$$

$$= -\frac{1}{n\beta d}\int_0^{-n\beta d} \left|\frac{\sin z}{z}\right|^2 dz$$

$$= \frac{1}{n\beta d}\int_0^{n\beta d} \left|\frac{\sin z}{z}\right|^2 dz$$

For a large element array,

$$n \to \infty$$

$$\therefore n\beta d \to \infty$$

$$U_{avg} = \frac{1}{n\beta d}\int_0^\infty \left|\frac{\sin z}{z}\right|^2 dz$$

Since $\int_0^\infty \left|\frac{\sin z}{z}\right|^2 dz = \frac{\pi}{2}$, we get

$$U_{avg} = \frac{1}{n\beta d} \times \frac{\pi}{2} = \frac{\pi}{2n\beta d}$$

$\therefore$ Directivity $D = \frac{U_{max}}{U_{avg}}$.

At $\theta = 0°$, $U_{max} = 1$.

Substituting, $D = \frac{1}{\frac{\pi}{2n\beta d}} = \frac{2n\beta d}{\pi}$

We know that $\beta = \frac{2\pi}{\lambda}$,

$$\therefore D = \frac{2 \times n \times 2\cancel{\pi} d}{\cancel{\pi}\lambda} = \frac{4nd}{\lambda}$$

$$\therefore \boxed{D = \frac{4nd}{\lambda}}$$

We know that, length of array $L = (n-1)d$.

$L \simeq nd$

$$\therefore \boxed{D = \frac{4L}{\lambda}}$$

Dolph - Tchebysheff Antenna

Design Procedure:

1. Determine the value of r, where

$$r = \frac{\text{main lobe Max}}{\text{Side lobe level}}$$

Side lobe level below main lobe maximum

in dB $= 20 \log_{10} r$

2. Find x_0.

Tchebysheff polynomial has same degree as array polynomial.

ie; ~~Ec~~ $T_{(n-1)}(x_0) = r$

Solve for x_0 where

$$x_0 = \frac{1}{2}\left[\left(r + \sqrt{r^2 - 1}\right)^{1/m} + \left(r - \sqrt{r^2 - 1}\right)^{1/m}\right]$$

Where $m = n-1$

3. Choose array polynomial.

$$E_0 = a_0 + a_1\left[2z^2 - 1\right] + a_2\left[8z^4 - 8z^2 + 1\right] + \dots$$

for odd element array,

$$E_0 = a_0 + a_1 T_2(z) + a_2 T_4(z) + \dots$$

for even element array,

$$E_e = a_0 z + a_1(4z^3 - 3z) + a_2(16z^5 - 20z^3 + 5z) + \cdots$$

4. Equate Tchebysheff polynomial ~~with~~ $T_{n-1}(x)$ with array polynomial,

$$T_{n-1}(x) = E_t$$

and calculate the coefficients and ratio for relative amplitude.

$$FNBW = 2\sin^{-1}\left[\frac{\lambda}{\pi D}\cos^{-1}\left[\frac{1}{x_0}\cos\frac{\pi}{2(m-1)}\right]\right]$$

$\cosh^{-1}\frac{x}{\sqrt{2}}$

$$HPBW = 2\sin^{-1}\left[\frac{\lambda}{\pi D}\cos^{-1}\left[\frac{1}{x_0}\cosh\left(\frac{\cosh^{-1}\frac{x}{\sqrt{2}}}{m-1}\right)\right]\right]$$

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The directivity of linear broad side array can be optimized at a desired side lobe level for all side lobes using Dolph-Tchebysheff array. This means we can set any value of side lobe level at ~~low~~ high value of directivity.

Dolph-Tchebysheff array follows Tchebysheff polynomial ie;

$$T_m(x) = \begin{cases} \cos(m\cos^{-1}x) & \text{for } -1 < x < +1 \\ \cosh(m\cosh^{-1}x) & \text{for } |x| > 1 \end{cases}$$

$m = 0, 1, 2, 3 \ldots$

Let $\theta = \cos^{-1} x$ $\quad \theta = \psi/2$ $\quad \boxed{\cos 2\theta = 2\cos^2\theta - 1}$

$x = \cos\theta$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

When $m = 0$,

$T_0(x) = \cos(0 \cdot \cos^{-1} x) = \underline{\underline{1}}$

$m = 1$, $\quad T_1(x) = \cos(\cos^{-1} x) = \underline{\underline{x}}$

$m = 2$, $\quad T_2(x) = \cos(2 \underbrace{\cos^{-1} x}_{\theta}) = \cos(2\theta)$

$= 2\underbrace{\cos^2\theta}_{x} - 1$ $\qquad T_2(x) = 2x^2 - 1$

$T_2(x) = \underline{\underline{2x^2 - 1}}$

$m = 3$, $\quad T_3(x) = \cos(3\cos^{-1} x) = \cos(3\theta)$

$= 4\cos^3\theta - 3\cos\theta$

$T_3(x) = \underline{\underline{4x^3 - 3x}}$

NOTE:

$$\boxed{T_m(x) = 2x\,[T_{m-1}(x)] - T_{m-2}(x)}$$

$T_4(x) = 2x\,[T_3(x) - T_2(x)]$

$= 2x\,[4x^3 - 3x - (2x^2 - 1)]$

$= 2x\,[4x^3 - 3x \mp 2x^2 + 1]$

$= 8x^4 - 6x^2 \mp 4x^3 + 2x$

$$T_4(x) = 2x[T_3(x)] - T_2(x)$$
$$= 2x[4x^3 - 3x] - (2x^2 - 1)$$
$$= 8x^4 - 6x^2 - 2x^2 + 1$$
$$T_4(x) = 8x^4 - 8x^2 + 1$$

$$T_5(x) = 2x[T_4(x)] - T_3(x)$$
$$= 2x[8x^4 - 8x^2 + 1] - (4x^3 - 3x)$$
$$= 16x^5 - 16x^3 + 2x - 4x^3 + 3x$$
$$T_5(x) = 16x^5 - 20x^3 + 5x$$

$$T_6(x) = 2x[T_5(x)] - T_4(x)$$
$$= 2x[16x^5 - 20x^3 + 5x] - (8x^4 - 8x^2 + 1)$$
$$= 32x^6 - 40x^4 + 10x^2 - 8x^4 + 8x^2 - 1$$
$$T_6(x) = 32x^6 - 48x^4 + 18x^2 - 1$$

Q. Design a 4 element broad side array of $\lambda/2$ spacing between the elements. The pattern is to be optimum with side lobe level 19.1 dB down from the main lobe maximum.

Sometimes (-19.1 dB) down from main...

Same as 19.1 dB down.

Sol: Given,

13 $\Rightarrow$ $19.1 = 20 \log x$

$20 \log x = 19.1$

$\log x = \frac{19.1}{20}$, $x = \text{Antilog}\left(\frac{19.1}{20}\right)$

$x = 9.015$

$\therefore \underline{\underline{x \simeq 9}}$

$n = 4.$

$m = n - 1 = 4 - 1$

$\underline{\underline{m = 3}}$

$$\Rightarrow x_0 = \frac{1}{2}\left[\left(x + \sqrt{x^2 - 1}\right)^{1/m} + \left(x - \sqrt{x^2 - 1}\right)^{1/m}\right]$$

$$= \frac{1}{2}\left[\underbrace{\left(9 + \sqrt{80}\right)^{\frac{1}{3}}}_{2.618033989} + \left(9 - \sqrt{80}\right)^{\frac{1}{3}}\right]$$

$$= \underline{\underline{1.5}}$$

$$\Rightarrow E_e = \underbrace{a_0 T_1(z) + a_1 T_3(z)} + a_2 T_5(z) + \ldots\ldots$$

[Since 4 elements]

$$E_4 = a_0 z + a_1 (4z^3 - 3z)$$

$\Rightarrow$ Equate array polynomial with Tchebyscheff polynominal.

$$T_{n-1}(x) = E_4 \Rightarrow E_4 = T_{(n-1)} x = E_4 = T_3(x)$$

$$a_0 z + a_1 (4z^3 - 3z) = 4x^3 - 3x$$

Replace $z = \frac{x}{x_0}$

$t = \frac{x}{x_0}$

$$a_0 \frac{x}{x_0} + a_1\left(4\left(\frac{x}{x_0}\right)^3 - \frac{3x}{x_0}\right) = 4x^3 - 3x.$$

Equate coefficients of x,

$$x\left[\frac{a_0}{x_0} - \frac{3a_1}{x_0}\right] = -3\cancel{x}$$

$$a_0 - 3a_1 = -3x_0 \qquad (x_0 = 1.5)$$

$$a_0 - 3a_1 = -3 \times 1.5$$

$$a_0 - 3a_1 = -4.5 \quad \text{—— (1)}$$

Equate coefficients of x^3,

$$\cancel{x^3}\left[\frac{4a_1}{x_0^3}\right] = 4\cancel{x^3}$$

$$\frac{\cancel{4}a_1}{x_0^3} = \cancel{4}$$

$$\therefore a_1 = x_0^3 = (1.5)^3$$

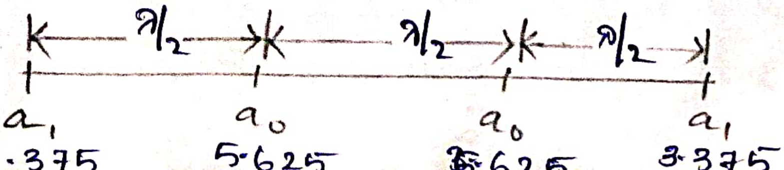
$$a_1 = 3.375 \quad \text{—— (2)}$$

Substitute (2) in (1);

$$a_0 - 3(3.375) = -4.5$$

$$a_0 - 10.125 = -4.5 \Rightarrow a_0 = -4.5 + 10.125$$

$$\underline{a_0 = 5.625}$$

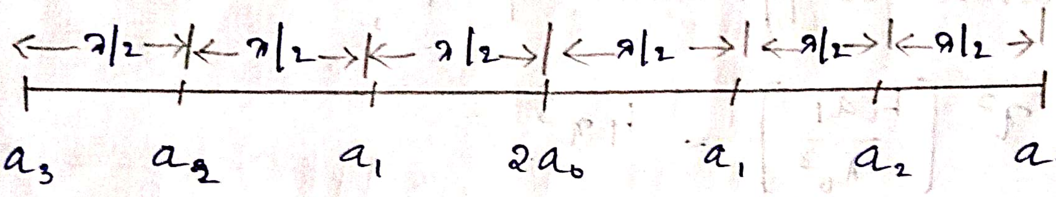


$$\frac{a_1}{a_1} : \frac{a_0}{a_1} : \frac{a_0}{a_1} : \frac{a_1}{a_1}$$

$$\frac{3.375}{3.375} : \frac{5.625}{3.375} : \frac{5.625}{3.375} : \frac{3.375}{3.375}$$

$$1 : 1.67 : 1.67 : 1$$

Q Design a 7 element broad side array which has optimum pattern for a side level lobe level of -20 dB. The spacing between the elements has to be $\lambda/2$.



$$\frac{a_3}{a_3} : \frac{a_2}{a_3} : \frac{a_1}{a_3} : \frac{2a_0}{a_3} : \frac{a_1}{a_3} : \frac{a_2}{a_3} : \frac{a_3}{a_3}$$

Given, $n = 7$

$$20 \log r = 20$$

$$\log r = \frac{20}{20}$$

$$r = 10$$

$$m = n - 1$$
$$= \underline{\underline{6}}$$

$$x_0 = \frac{1}{2}\left[\left(x+\sqrt{x^2-1}\right)^{\frac{1}{m}} + \left(x-\sqrt{x^2-1}\right)^{\frac{1}{m}}\right]$$

$$= \frac{1}{2}\left[\left(10+\sqrt{998}\right)^{\frac{1}{6}} + \left(10-\sqrt{998}\right)^{\frac{1}{6}}\right]$$

$$= \underline{\underline{1.127}}$$

$$E_0 = a_0 + a_1 T_2(z) + a_2 T_4(z) + a_3 T_6(z)$$

$$E_7 = a_0 + a_1 T_2(z) + a_2 T_4(z) + a_3 T_6(z)$$

$$= a_0 + a_1(2z^2-1) + a_2(8z^4 - 8z^2 + 1) + a_3(32z^6 - 48z^4 + 18z^2 - 1)$$

Equate polynomial with Tchebysheff polynomial,

$$T_6(x) = E_7$$

$$E_7 = T_6(x)$$

$$a_0 + a_1(2z^2-1) + a_2(8z^4 - 8z^2 + 1) + a_3(32z^6 - 48z^4 + 18z^2 - 1)$$

$$= 32x^6 - 48x^4 + 18x^2 - 1$$

Replace z by $\frac{x}{x_0}$

$$a_0 + a_1\left[2\left(\frac{x}{x_0}\right)^2 - 1\right] + a_2\left[8\left(\frac{x}{x_0}\right)^4 - 8\left(\frac{x}{x_0}\right)^2 + 1\right] +$$

$$a_3\left[32\left(\frac{x}{x_0}\right)^6 - 48\left(\frac{x}{x_0}\right)^4 + 18\left(\frac{x}{x_0}\right)^2 - 1\right]$$

$$= 32x^6 + 48x^4 + 18x^2 - 1$$

Equate coefficients of x^2,

$$\cancel{x^2}\left[\frac{2a_1}{x_0^2} - \frac{8a_2}{x_0^2} + \frac{18a_3}{x_0^2}\right] = 18\cancel{x^2}$$

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$$= \frac{2a_1 - 8a_2 + 18a_3}{x_0^2} = 18$$

$$= 2a_1 - 8a_2 + 18a_3 = 18(x_0^2)$$

$$2a_1 - 8a_2 + 18a_3 = 22.867 \quad \text{—— (1)}$$

Equate coefficients of x^4,

$$\cancel{x^4}\left[\frac{8a_2}{x_0^4} + \frac{48a_3}{x_0^4}\right] = -48\cancel{x^4}$$

$$\frac{8a_2 + 48a_3}{x_0^4} = -48$$

$$8a_2 + 48a_3 = -48(x_0^4)$$

$$8a_2 + 48a_3 = -77.43 \quad \text{—— (2)}$$

Equate coefficients of x^6,

$$\cancel{x^6}\left[\frac{32a_3}{x_0^6}\right] = 32\cancel{x^6}$$

$$\cancel{32}a_3 = \cancel{32}(x_0^6)$$

$$a_3 = 2.049$$

$$a_3 = 2.05 \quad \text{—— (3)}$$

(3) in (2)

$$8a_2 - 48(2.05) = -77.43$$

$$8a_2 = (-77.43 + 98.4)/8$$

$$a_2 = 2.62$$

Substitute a_2 and a_3 in 1

$2a_1 - 8(2.62) + 18(2.05) = 22.86$

$2a_1 - 20.96 + 36.9 = 22.86$

$2a_1 = 6.92$

$a_1 = \frac{6.92}{2} = \underline{\underline{3.46}}$

Equating constants,

$a_0 - a_1 + a_2 - a_3 = -1$

$a_0 - (3.46) + (2.62) - (2.05) = -1$

$a_0 - 2.89 = -1$

$a_0 = \underline{1.89}$

a_3	a_2	a_1	$2a_0$	a_1	a_2	a_3
2.05	2.62	3.46	3.78	3.46	2.62	2.05

$\frac{a_3}{a_3} : \frac{a_2}{a_3} : \frac{a_1}{a_3} : \frac{2a_0}{a_3} : \frac{a_1}{a_3} : \frac{a_2}{a_3} : \frac{a_3}{a_3}$

$\frac{2.05}{2.05} : \frac{2.62}{2.05} : \frac{3.46}{2.05} : \frac{3.78}{2.05} : \frac{3.46}{2.05} : \frac{2.62}{2.05} : \frac{2.05}{2.05}$

$1 : 1.27 : 1.68 : 1.84 : 1.68 : 1.27 : 1$

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Module - 4

MICROWAVES

16

Microwaves are electromagnetic waves whose frequency ranging from 300 MHz - 300GHz

Q. Write the advantages and applications of Microwaves.

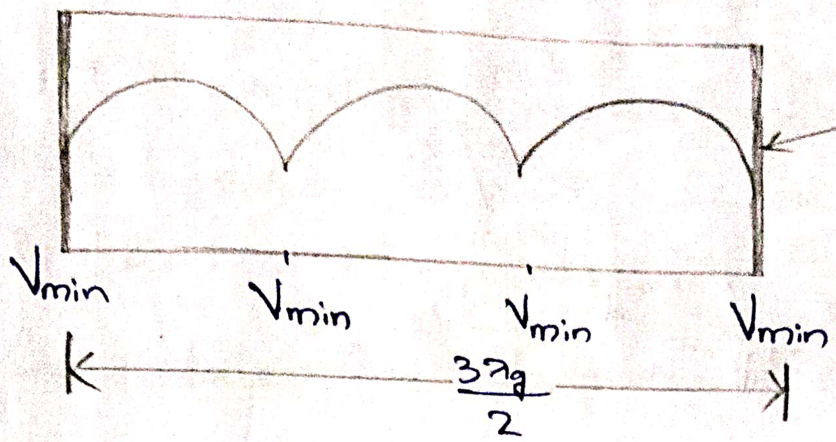
Advantages:

* High frequency operation
* Low latency
* Long-distance transmission
* Resistance to interference [from other electronic devices]

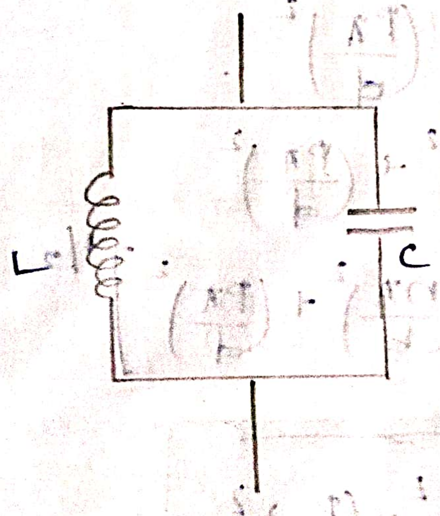
Applications:

* Cellular Networks (2G, 3G, 4G, 5G)
* Satellite Communications.
* Bluetooth technology
* Wireless Local Area Networks (WLANs)
* Microwave imaging
* Heating and drying materials
* Spectroscopy
* Radar Systems
* RFID tags

Cavity Resonator



When one end of the wave guide is terminated in a Shorting plate, there will be reflections and hence Standing waves as shown in figure. when another Shorting plate is kept at a distance of multiples of $\lambda g/2$, then the hollow Space So formed can Support a Signal which bounces back and forth between the two Shorting plates. This results in resonance and hence the hollow Space is called a ~~ca~~ cavity and the resonator as cavity resonator.



$$\boxed{f = \frac{1}{2\pi\sqrt{LC}}}$$

A microwave cavity resonator can be considered as a tuned circuit having resonant frequency $f = \frac{1}{2\pi\sqrt{LC}}$

Rectangular Cavity Resonator.

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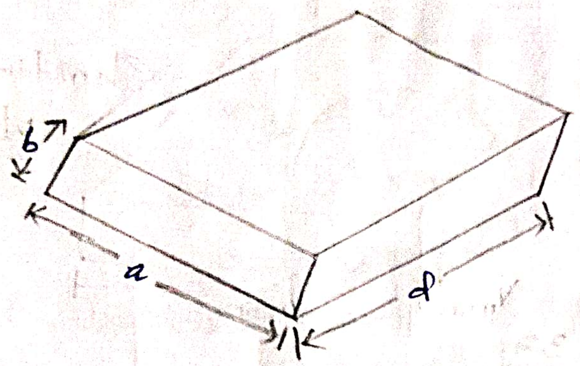


$h^2 = \gamma^2 + \omega^2\mu$

For the rectangular wave guide,

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \gamma^2$$

$$\left[\begin{array}{l}\gamma = j\beta \\ \gamma^2 = -\beta^2\end{array}\right.$$

$$\omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \beta^2$$

Inorder to resonate the rectangular wave guide has to satisfy the condition, $\beta = \frac{p\pi}{d}$

$$\therefore \omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$(2\pi f_0)^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2$$

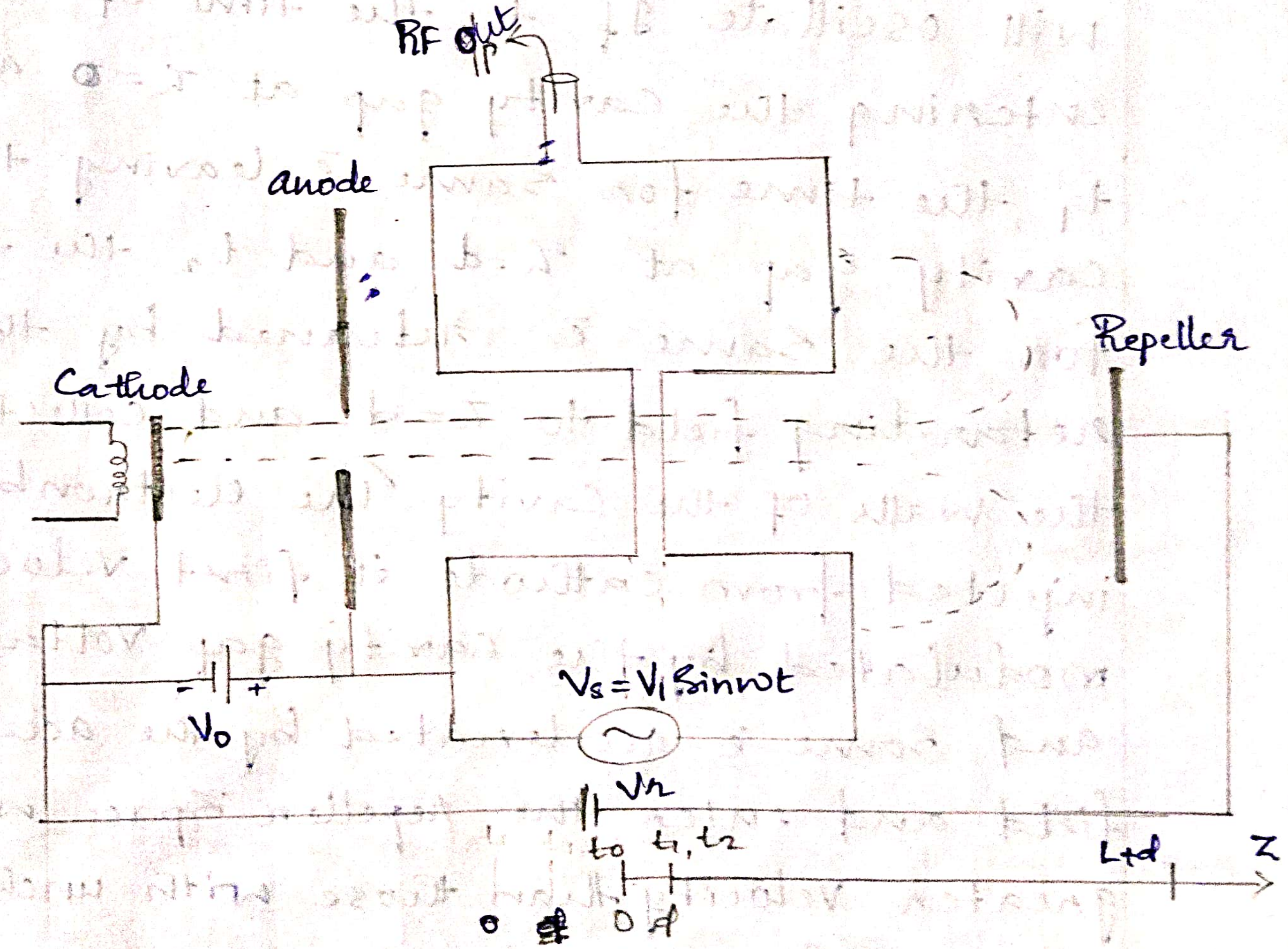
$$f_0 = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]^{1/2}$$

$$\boxed{f_0 = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}}$$

Application of Cavity Resonator:

* These are used as tuned circuits in
  1. Ultra High Frequency [UHF] tubes
  2. Klystron amplifiers/oscillators
  3. Cavity Magnetrons.
* ~~In dule.~~ In duplexers of radars.
* Cavity wave meters in measurement of frequency.

124 Reflex Klystron Oscillator.

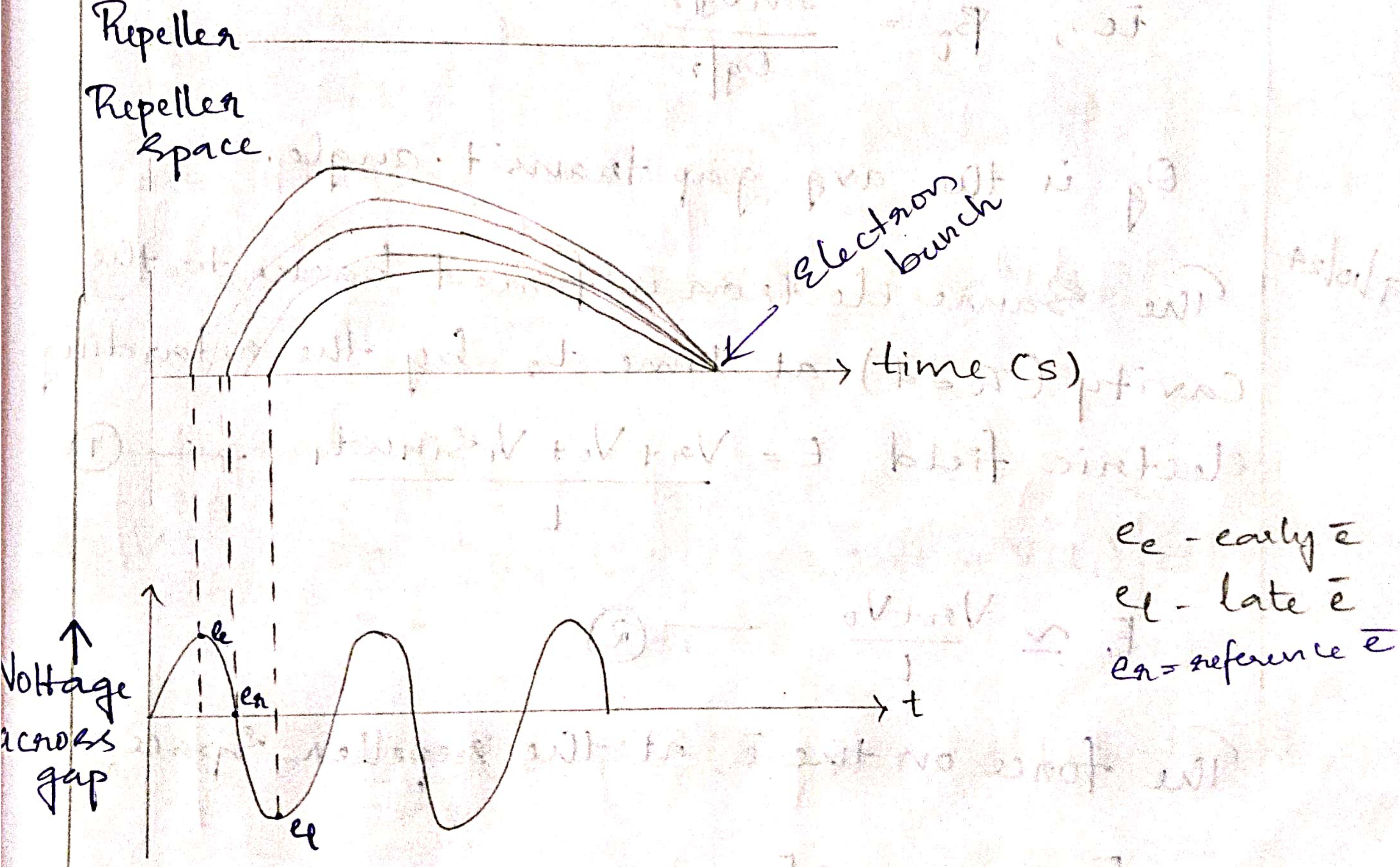


18 It is low power low efficiency microwave oscillator used as signal source in microwave generator; Local oscillators in microwave receivers etc and its o/p power ranges from 10mW to 3W and frequency ranges from 4GHz-200GHz.

If a fraction of o/p power is fed back to the i/p cavity and the loop gain has a magnitude of unity with a phase shift of multiple of 2π, then the klystron will oscillate. If t_0, the time of $\bar{e}$ entering the cavity gap at $z=0$ and t_1, the time for same $\bar{e}$ leaving the cavity gap at $z=d$ and t_2 the time for the same $\bar{e}$ returned by the retarding field to $z=d$ and collected on the walls of the cavity. The electron beam injected from cathode is first velocity modulated by the cavity gap voltage and some $\bar{e}$ accelerated by the accelerating field and enter the repeller space with greater velocity than those with unchanged velocity. Some $\bar{e}$ decelerated by the retarding field enter the cavity with less velocity.

All $\bar{e}$ returned by the repeller voltage passes through the cavity in bunches. On the return journey, the bunched $\bar{e}$s give up their kinetic energy to the electromagnetic field in the cavity. Then the $\bar{e}$s are finally collected by the walls of the cavity.

Apple-Gate Diagram.



The electrons entering the cavity gap from the cathode at $z=0$ at time t_0 with a constant velocity,

$$V_0 = \sqrt{\frac{2eV_0}{m}}$$

$$= 0.593 \times 10^6 \sqrt{V_0}$$

19 The Same $\bar{e}$ leaves the cavity at $z = d$ at time t_1 with velocity $v(t_1)$

$$v(t_1) = V_0 \left[1 + \frac{\beta_i V_1}{2V_0} \sin\left(\omega t_1 - \frac{\theta_g}{2}\right) \right]$$

Where β_i is the beam coupling coefficient and is the ratio of alternating current induced in the cavity to the alternating component of beam current that produces it.

ie., $\beta_i = \dfrac{\sin \theta_g/2}{\theta_g/2}$

θ_g is the avg gap transit angle.

9/10/24 The Same electron is forced back to the cavity ($z = d$) at time t_2 by the retarding electric field $E = \dfrac{V_r + V_0 + V_1 \sin\omega t_1}{L}$ —— (1)

V_1 is very minute

$$E \simeq \frac{V_r + V_0}{L} \quad \text{—— (2)}$$

The force on the $\bar{e}$ at the repeller space,

$$F = ma = -eE$$

$$m\frac{d^2 z}{dt^2} = -e\left[\frac{V_r + V_0}{L}\right]$$

$$\frac{d^2 z}{dt^2} = \frac{-e[V_r + V_0]}{mL}$$

Integrating,

$$\frac{dz}{dt} = -\frac{e}{mL}(V_r+V_0)\int_{t_1}^{t} dt$$

$$\frac{dz}{dt} = -\frac{e}{mL}(V_r+V_0)\left[t\right]_{t_1}^{t} + \underbrace{k_1}_{\text{Integral constant}}$$

$$\frac{dz}{dt} = -\frac{e}{mL}(V_r+V_0)(t-t_1) + k_1 \quad \text{—— (3)}$$

At time $t = t_1$,

$$\frac{dz}{dt} = k_1 = V(t_1)$$

$$\Rightarrow \int_{t_1}^{t}(t-t_1)\,dt = \left[\frac{(t-t_1)^2}{2}\right]_{t_1}^{t}$$

Substituting this in (3)

$$(3) \Rightarrow \frac{dz}{dt} = -\frac{e}{mL}(V_r+V_0)(t-t_1) + V(t_1)$$

Integrating,

$$z = -\frac{e}{mL}(V_r+V_0)\int_{t_1}^{t}(t-t_1)\,dt + V(t_1)\int_{t_1}^{t} dt$$

$$z = -\frac{e}{2mL}(V_r+V_0)(t-t_1)^2 + V(t_1)(t-t_1) + \underbrace{k_2}_{\text{constant}} \quad \text{—— (4)}$$

at $t = t_1$, $z = k_2 = d$

Substituting,

$$(4) \Rightarrow z = -\frac{e}{2mL}(V_r+V_0)(t-t_1)^2 + V(t_1)(t-t_1) + d$$

At time t_2 and $x = d$, the $\bar{e}$ will re-enter the cavity.

$$\cancel{d} = -\frac{e}{2mL}(V_r+V_0)(t_2-t_1)^2 + v(t_1)(t_2-t_1) + \cancel{d}$$

$$\cancel{-}\,v(t_1)(t_2-t_1) = \cancel{-}\frac{e}{2mL}(V_r+V_0)(t_2-t_1)^2$$

$$v(t_1) = \frac{e}{2mL}(V_r+V_0)(t_2-t_1)$$

$$(t_2-t_1) = \frac{2mL\,v(t_1)}{e(V_r+V_0)}$$

Substituting the value of $v(t_1)$,

$$(t_2-t_1) = \frac{2mL\,V_0\left[1+\frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1-\frac{\theta_g}{2}\right)\right]}{e(V_r+V_0)}$$

$$(t_2-t_1) = \underbrace{\frac{2mL}{e(V_r+V_0)}}_{T_0'} V_0\left[1+\frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1-\frac{\theta_g}{2}\right)\right] \quad (5)$$

(ii) $$\boxed{(t_2-t_i) = T_0'\left[1+\frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1-\frac{\theta_g}{2}\right)\right]}$$

where $T_0' = \dfrac{2mL\,V_0}{e(V_r+V_0)}$ The round-trip DC transit time

Multiply by ω,

$$w(t_2 - t_1) = \omega T_0' \left[1 + \frac{\beta_i V_1}{2V_0} \sin(\omega t_1 - \theta_g/2)\right]$$

$$T' = \theta_0' \left[1 + \frac{\beta_i V_1}{2V_0} \sin(\omega t_1 - \theta_g/2)\right]$$

Where $\theta_0' = \omega T_0'$, $T' = w(t_2 - t_1)$

$$T' = \theta_0' + \theta_0' \frac{\beta_i V_1}{2V_0} \sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\Big]$$

$$T' = \theta_0' + x' \sin\left(\omega t_1 - \frac{\theta_g}{2}\right)$$

where $\underbrace{x' = \theta_0' \frac{\beta_i V_1}{2V_0}}_{\text{Bunching Parameter.}}$

Power output Efficiency

The round trip transit time

$T' = (t_2 - t_1)$ and the round trip transit angle, $\omega T_0' = \theta_0'$.

The maximum bunching occurs when,

$\theta_0' = 2n\pi - \pi/2$ where n is the no. of cycles.

The magnitude of the fundamental current component, $I_2 = 2 I_0 \beta i J_1(x')$ $2I_0 \beta_i J_1(x')$

The DC power supply by the beam voltage,

$P_{dc} = V_0 I_0$

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The AC power delivered to the load,

$$P_{ac} = \frac{V_1 I_2}{2}$$

$$= \frac{V_1 \, 2 I_0 \beta_i J_1(x')}{2}$$

$\theta_0' = 2n\pi - \pi/2$ (for max. ban)

$$P_{ac} = V_1 I_0 \beta_i J_1(x') \quad \text{——— (1)}$$

We know the bunching parameter

$$x' = \frac{\theta_0' \beta_i V_1}{2V_0}, \quad V_1 = \frac{2V_0 x'}{\beta_i \theta_0'}$$

$$V_1 = \frac{2V_0 x'}{\beta_i (2n\pi - \pi/2)} \quad \text{——— (2)}$$

Substitute eqn (2) in (1)

$$(1) \Rightarrow \quad P_{ac} = \frac{2V_0 x' \; I_0 \cancel{\beta_i} J_1(x')}{\cancel{\beta_i}(2n\pi - \pi/2)}$$

$$P_{ac} = \frac{2V_0 x' I_0 J_1(x')}{(2n\pi - \pi/2)}$$

The electronic efficiency of reflex klystron,

$$\eta = \frac{P_{ac}}{P_{dc}} = \frac{2 \cancel{V_0} x' \cancel{I_0} J_1(x')}{(2n\pi - \pi/2) \cancel{V_0} \cancel{I_0}}$$

$$\eta = \frac{2x' J_1(x')}{2n\pi - \pi/2}$$

When $x = 2.408$ and $n = 2$, the maximum o/p is delivered to the load. Therefore the first order Bessel function value,

$$J_1(x') = 0.52$$

$$\eta = \frac{2 \times 2.408 \times 0.52}{4\pi - \pi/2} = 0.227$$

$$= \underline{\underline{22.7\%}}$$

17/10/24

Admittance:

The induced current in phasor form,

$$i_2 = 2I_0 \beta_i J_1(x') e^{-j\theta_0}$$

The Voltage across the gap at time t_2 in phasor form,

$$V_2 = V_1 e^{-j\pi/2}$$

So, the electronic admittance of klystron Oscillator is the ratio of I_2, i_2 and V_2.

Admittance, $Y_e = \frac{i_2}{V_2}$

$$\frac{i_2}{V_2} = \frac{2I_0 \beta_i J_1(x') e^{-j\theta_0}}{V_1 e^{-j\pi/2}} \quad \text{——} \; (1)$$

We know the bunching parameter x_1' is

$$x' = \frac{V_1 \beta_i \theta_0'}{2V_0}$$

$$V_1 = \frac{2V_0 x'}{\beta_i \theta_0'} \quad \text{--- (2)}$$

Substitute (2) in (1);

$$Y_e = \frac{2I_0 \beta_i J_1(x') e^{-j\theta_0}}{\frac{2V_0 x'}{\beta_i \theta_0'} e^{-j\pi/2}}$$

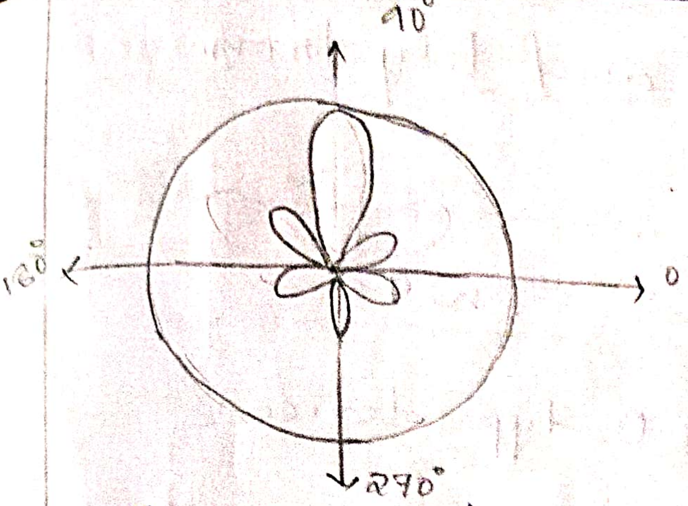
$$= \frac{I_0}{V_0} \times \frac{\beta_i^2 \theta_0'}{2} \times \frac{2J_1(x')}{x'} \times e^{j(\pi/2 - \theta_0)}$$

$$\boxed{Y_e \propto \frac{2J_1(x')}{x'}}$$

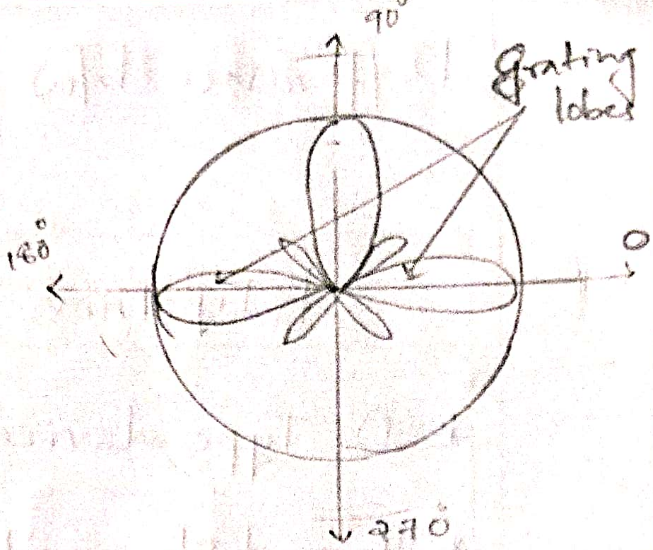
Grating lobes: [MOD: 3 Antenna portion]

In a uniformly spaced array, it is possible that the array having equally strong radiation in the direction other than major lobes direction. These unintended beams of radiations are known as grating lobes.

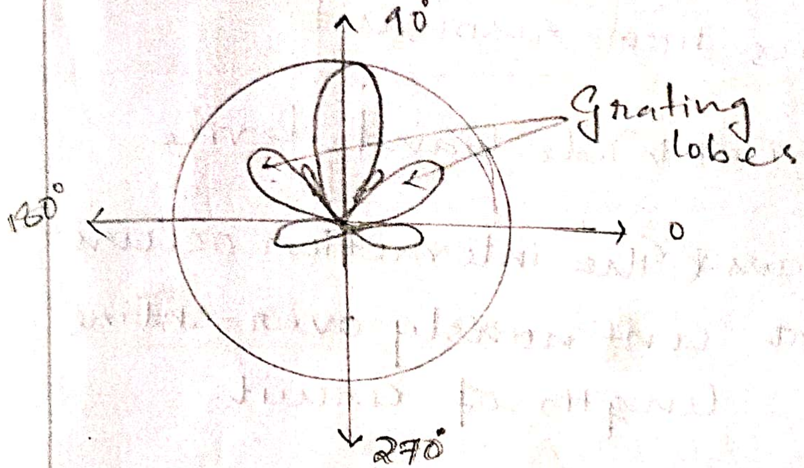
They occur in uniformly spaced arrays when the spacing between the elements is greater than $\lambda/2$. Grating lobes can be eliminated by restricting the spacing upto $\lambda/2$.



Spacing = λ/2



Spacing = λ



Spacing = 1.5 λ

Travelling Wave Tube:

A travelling wave tube [TWT] consist of electron beam and slow-wave structures. The ē s are focused by constant magnetic field along the electron beam.

23. Difference b/w TWT and klystron.

Klystron	TWT
* O-Type device	* O-type device
* I/p and O/p cavities are resonant cavities.	* I/p and O/p circuits are non-resonant.
* Narrow-band device	* Wide-band device
* The interaction occurs at ~~fre~~ few resonant cavities.	* The interaction occurs continously over entire lengths of circuit.
* Non-propagating wave	* Propagating wave
* Each cavity operates independently	* Each cavity operates dependently.

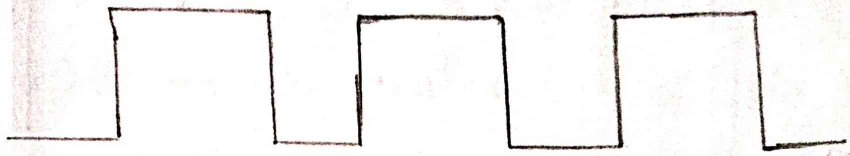
Slow wave Structures.

Slow wave structures reduces the wave velocity, in a certain direction so that the electron beam and signal wave can interact. It is ~~kt~~ non-resonant type

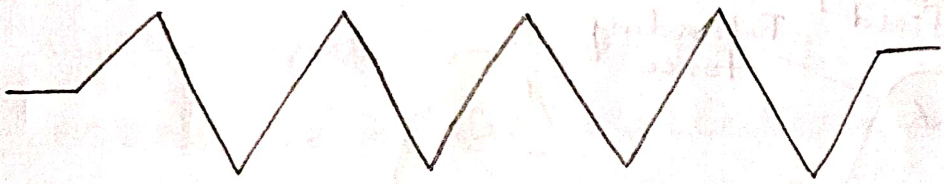
Types:

1. Helical line.

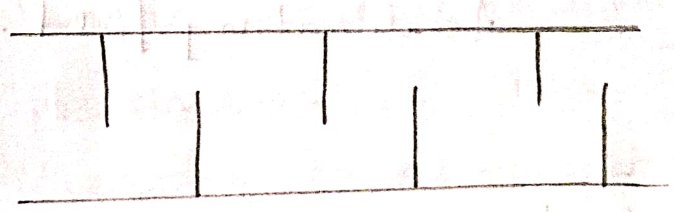
2. Folded-back line.



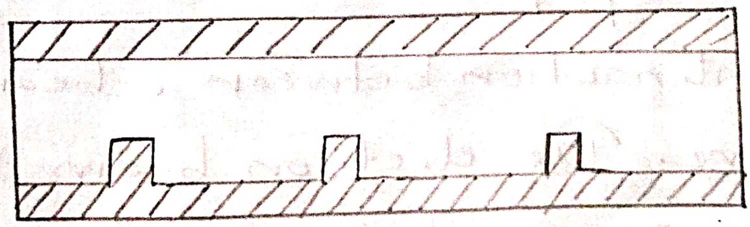
3. Zig Zag line:



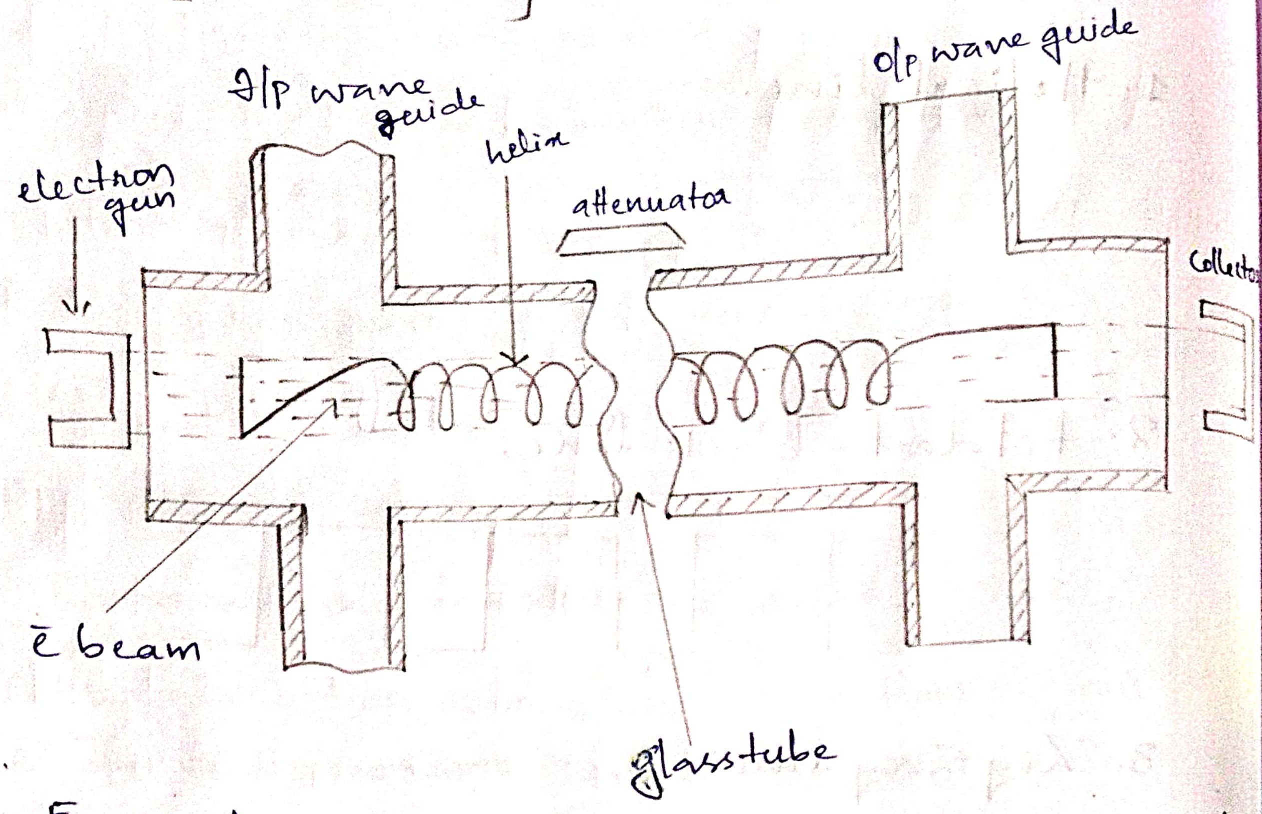
4. Inter-Digital line:

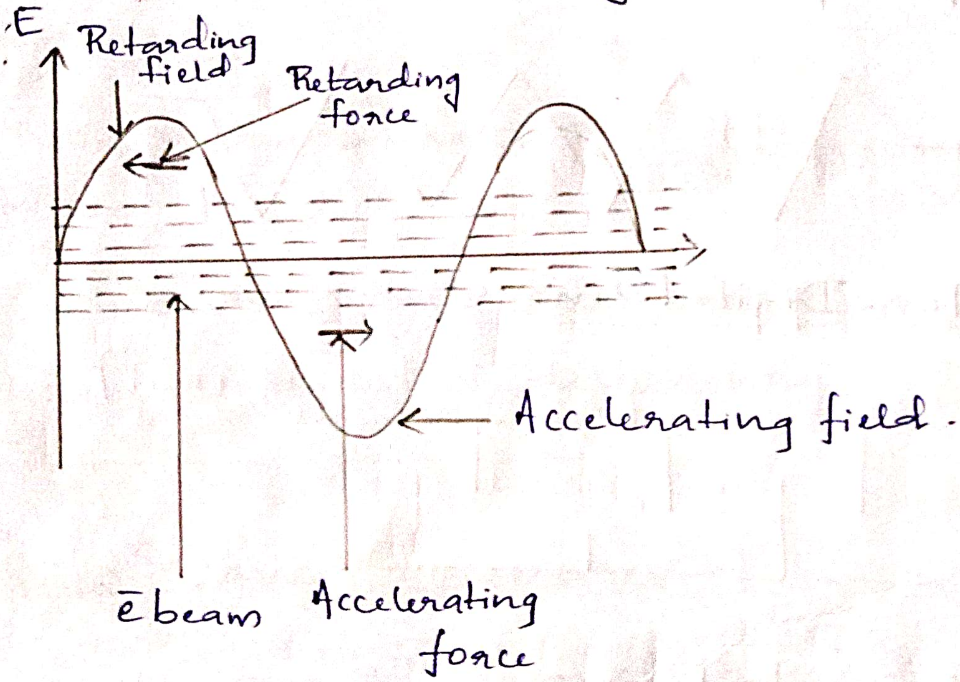


5. Corrugated line.



TWT [Structure]





Working:

TWT is an amplifier which makes use of distributed interaction between ē beam and travelling wave. The electron beam travels with a velocity governed by anode voltage which is typically 2-10% of the velocity of electromagnetic wave in free space. The interaction b/w RF field and moving $\bar{e}$ will takes place only when the velocity of RF field is retarded by some means. This is done by slow-wave structures.

TWT uses a helix as slow-wave structure. There exist spatially periodic travelling wave coaxial to the helix and the $\bar{e}$ beam is accelerated to a velocity which is slightly more than the phase velocity of axial wave. The axial wave accelerates the $\bar{e}$ during one half cycle and deccelerates during second half cycle. But at any point of time, there exist more $\bar{e}$s in the deccelerating half cycle resulting in net transfer of energy from electrons to wave. The strengthened wave offers more decceleration to the incoming $\bar{e}$ and increasing $\bar{e}$ concentration in this region and thereby increasing energy transfer. It results in exponential growth of signal along the length of helix.

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Q. Show that the magnitude of velocity function of electron beam is directly proportional to the magnitude of axial electric field $[V_e \propto E]$

Sol: When the signal voltage is coupled into the helix, the axial field exerts a force on electrons,

$$F = -eE \quad \text{—— (1)}$$

If the travelling wave is propagating in z-direction then the z component of electric field can be expressed as,

$$E_z = E_1 \sin(\omega t - \beta_p z) \quad \text{—— (2)}$$

where β_p is the axial phase constant.

$$\beta_p = \frac{\omega}{V_p}, \quad V_p \text{ is the phase velocity.}$$

So the equation of electron motion from eqn (1) ⇒

$$m \cdot \frac{dv}{dt} = -e\left[E_1 \sin(\omega t - \beta_p z)\right] \quad \text{—— (3)}$$

Assume the velocity of electron,

$$V = V_0 + V_e \cos(\omega_e t + \theta_e) \quad \text{—— (4)}$$

where V_0 is the DC $\bar{e}$ velocity

V_e is the magnitude of velocity ~~function~~ fluctuation

θ_e is the phase angle of fluctuation

ω_e is the angular frequency of velocity fl..

$$\frac{dv}{dt} = -V_e \sin(\omega_e t + \theta_e) \cdot \omega_e$$

$$\frac{dv}{dt} = -V_e \omega_e \sin(\omega_e t + \theta_e) \quad \text{—— (5)}$$

Substitute eqn (5) in (3)

$$\rightarrow -mV_e\omega_e \sin(\omega_e t + \theta_e) = -e\left[E_1 \sin(\omega t - \beta_p z)\right]$$

$$V_e \sin(\omega_e t + \theta_e) = \frac{eE_1}{m\omega_e} \sin(\omega t - \beta_p z) \quad \text{—— (6)}$$

The distance travelled by the electron beam in z-direction,

$$z = V_0 (t - t_0)$$

$$V_e \sin(\omega_e t + \theta_e) = \frac{eE_1}{m\omega_e}\left[\sin(\omega t - \beta_p V_0 (t - t_0))\right]$$

$$= \frac{eE_1}{m\omega_e} \sin(\omega t - \beta_p V_0 t + \beta_p V_0 t_0)$$

$$V_e \sin(\omega_e t + \theta_e) = \frac{eE_1}{m\omega_e} \sin\left[(\omega - \beta_p V_0)t - \beta_p V_0 t_0\right] \quad \text{—— (7)}$$

Compare the terms on LHS and RHS,

$$V_e = \frac{eE_1}{m\omega_e} \rightarrow \boxed{V_e \propto E_1}$$

Velocity is directly proportional to axial field.

$\theta_e = \beta_p V_0 t_0$

$\omega_e = \omega - \beta_p V_0$

Here $\omega = \beta_p V_p$

$= \beta_p V_p - \beta_p V_0$

$$\boxed{\omega_e = \beta_p (V_p - V_0)}$$

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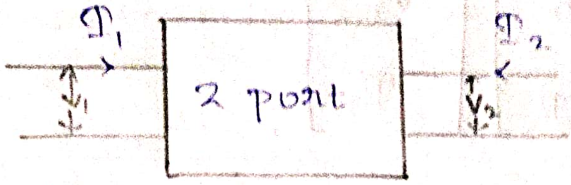
Module-5
MICROWAVE HYBRID CIRCUITS

Two port network:



For H parameter,

$$H \begin{cases} V_1 = h_{11} I_1 + h_{12} V_2 \\ I_2 = h_{21} I_1 + h_{22} V_2 \end{cases}$$

$$h_{11} = \frac{V_1}{I_1} \Big|_{V_2 = 0}$$

$$h_{12} = \frac{V_1}{V_2} \Big|_{I_1 = 0}$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$y \begin{cases} I_1 = y_{11} V_1 + y_{12} V_2 \\ I_2 = y_{21} V_1 + y_{22} V_2 \end{cases}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$z \begin{cases} V_1 = z_{11} I_1 + z_{12} I_2 \\ V_2 = z_{21} I_1 + z_{22} I_2 \end{cases}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

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ABCD $\begin{cases} V_1 = AV_2 - BI_2 \\ I_1 = CV_2 - DI_2 \end{cases}$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

All these network parameters depends on the total voltages and currents at each port. Thus from H parameter model,

$$h_{11} = \frac{V_1}{I_1} \Big|_{V_2=0}$$

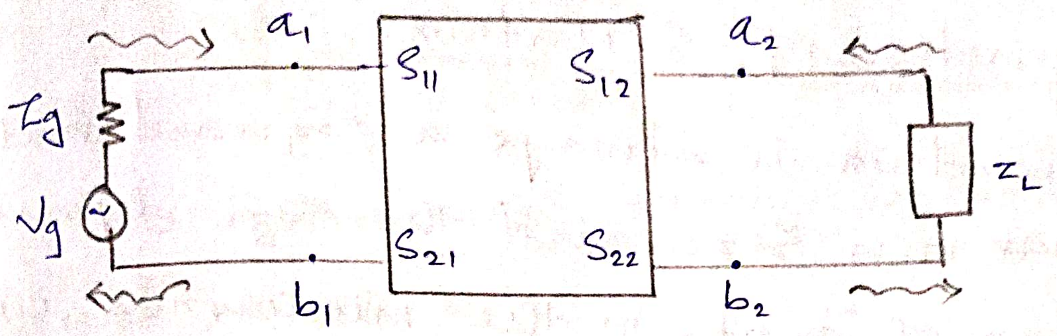
$$h_{12} = \frac{V_1}{V_2} \Big|_{I_1=0}$$

If the frequencies are in microwave range x, y, z and ABCD parameters cannot be measured due to

* the equipments are not readily available to measure the total voltages and currents at the ~~points~~ ports of the network.
* the short and open circuit are difficult to achieve over a broad range of frequencies.
* active devices such as power-transistors channel diodes etc will not have

Stability for a short and open circuit.

To overcome these difficulties S-matrix introduced in which the logical variables are travelling wave rather than voltages and currents in the network.



S-parameter two port n/w.

$$b_1 = S_{11} a_1 + S_{12} a_2$$

$$b_2 = S_{21} a_1 + S_{22} a_2$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

The Scattering matrix [S-matrix] of an n port junction is a square matrix of a set of element which relate the incident and reflected waves at the ports of the junction.

An S-parameter matrix the diagonal elements represents reflection coefficient and the off diagonal elements represents the transmission coefficients

Thus for a three port junction the S-matrix can be

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

Parameters of S-Matrix.

* S-matrix is always a square matrix of ~~max~~ nxn size and the size depends on the no. of ports in the microwave circuit.

* **Zero property:**

$$S_{11} S_{12}^* + S_{21} S_{22}^* + S_{31} S_{32}^* = 0$$

$$S_{11} S_{21}^* + S_{12} S_{22}^* + S_{13} S_{23}^* = 0$$

$$\boxed{\sum_{i=1}^{n} S_{ik} S_{ij}^* = 0}$$

The sum of product of each term of any row or colounm multiplied by the complex conjugate of the ~~components~~ corresponding terms of any other row and colounm will be equal to zero.

* **Unity property:**

$$S_{11} S_{11}^* + S_{21} S_{21}^* + S_{31} S_{31}^* = 1$$

$$S_{13} S_{13}^* + S_{23} S_{23}^* + S_{33} S_{33}^* = 1$$

$$\boxed{\sum_{i=1}^{n} S_{ij} S_{ij}^{*} = 1}$$

The sum of product of each term of any row or coloumn multiplied by its own complex conjugate is equal to unity.

* **Symmetric property:**

The symmetric terms in S-matrix are all equal that is,

$$S_{12} = S_{21} \; ; \; S_{13} = S_{31} \; ; \; S_{23} = S_{32}$$

ie ; $\boxed{S_{ij} = S_{ji}}$

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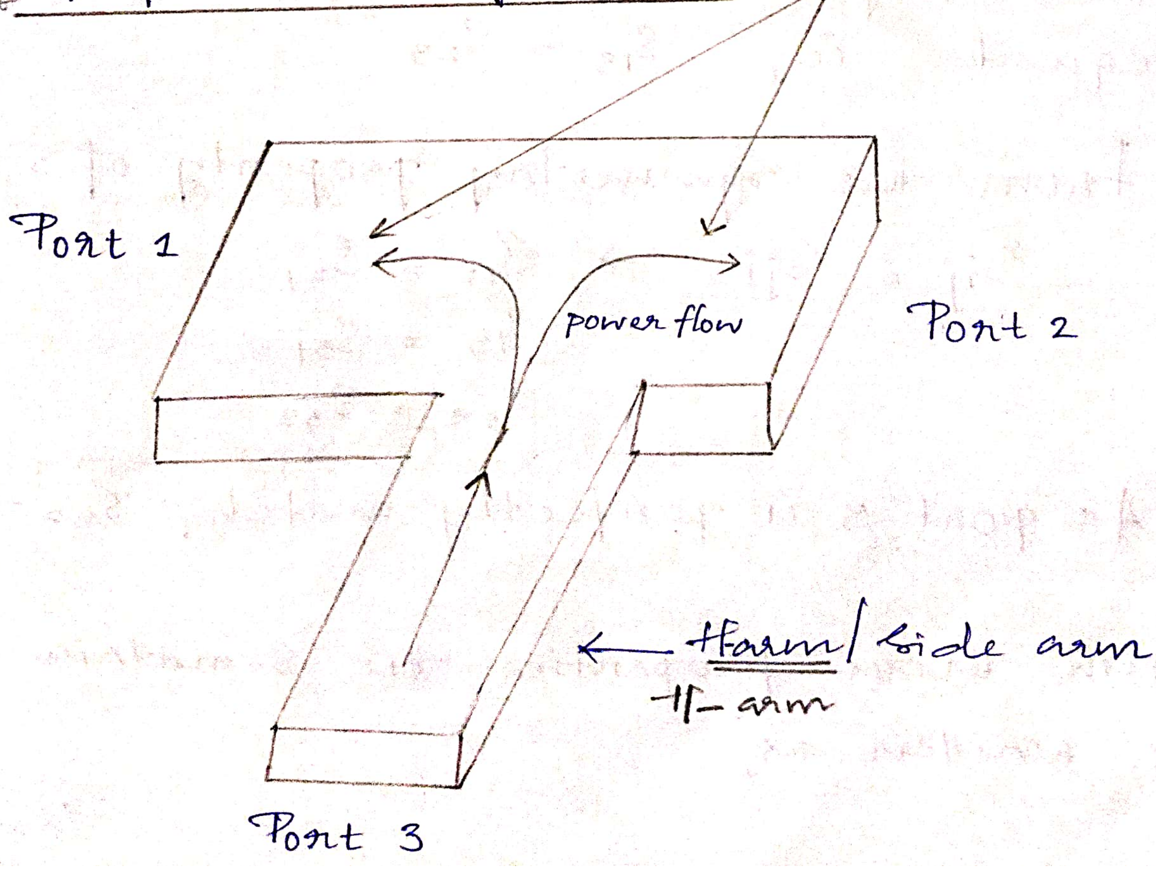
Microwave Components:

Tee Junction

* **H. plane Tee junction.** Coplanar arm.



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A tee junction is a intersection of three waveguides in the form of English alphabet T. H-plane tee is formed by cutting a rectangular slot along the width of main waveguide and attaching another waveguide or side-arm. H-plane tee is so called because the side arm is placed in the plane of magnetic field.

As it is a three-port n/w S matrix

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

* Because of the plane of symmetry, the scattering coefficients S_{13} and S_{23} are equal. ie; $S_{13} = S_{23}$

* From the symmetry property of S-matrix,

$$S_{ij} = S_{ji} \rightarrow \begin{aligned} S_{12} &= S_{21} \\ S_{13} &= S_{31} \\ S_{23} &= S_{32} \end{aligned}$$

* As port 3 is perfectly match, $S_{33} = 0$

With these properties, the S-matrix can be written as,

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix}$$

From unity property, $[S][S^*] = [I]$

$$\Rightarrow \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1C_1 \Rightarrow \quad S_{11}S_{11}^* + S_{12}S_{12}^* + S_{13}S_{13}^* = 1$

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \quad \text{———} \quad (1)$$

$R_2C_2 \rightarrow \quad S_{12}S_{12}^* + S_{22}S_{22}^* + S_{13}S_{13}^* = 1$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \quad \text{———} \quad (2)$$

$R_3C_3 \rightarrow \quad S_{13}S_{13}^* + S_{13}S_{13}^* + 0 = 1$

$$|S_{13}|^2 + |S_{13}|^2 = 1 \quad \text{———} \quad (3)$$

$R_3C_1 \Rightarrow \quad S_{13}S_{11}^* + S_{13}S_{12}^* + 0 . S_{13}^* = 0$

$$S_{13}S_{11}^* + S_{13}S_{12}^* = 0 \quad \text{———} \quad (4)$$

From eqn (3); $\quad 2|S_{13}|^2 = 1$

$$|S_{13}|^2 = 1/2$$

$$S_{13} = \frac{1}{\sqrt{2}}$$

① = ② By equating ① & ②;

$\rightarrow$ $|S_{11}|^2 + \cancel{|S_{12}|^2} + \cancel{|S_{13}|^2} = \cancel{|S_{12}|^2} + |S_{22}|^2 + \cancel{|S_{13}|^2}$

30 $$|S_{11}|^2 = |S_{22}|^2$$

$$S_{11} = S_{22}$$

~~By Substituting the above values in eqn ①;~~

~~①~~ $\rightarrow$ from eqn ④;

$$S_{13} S_{11}^* + S_{13} S_{12}^* = 0 \qquad [S_{13} \neq 0]$$

$$S_{13}[S_{11}^* + S_{12}^*] = 0$$

Here, $S_{11}^* + S_{12}^* = 0$ $\qquad [\because S_{13} \neq 0]$

$$S_{11}^* = -S_{12}^*$$

$$\therefore S_{11} = -S_{12}$$

Substitute the above parameters in ①;

$$|S_{11}|^2 + |S_{11}|^2 + |S_{13}|^2 = 1$$

$S_{12} = -S_{11}$ & $|-S_{11}| = S_{11}$

$$|S_{11}|^2 + |S_{11}|^2 + \frac{1}{2} = 1$$

$$2|S_{11}|^2 = 1/2 \quad (1 - 1/2)$$

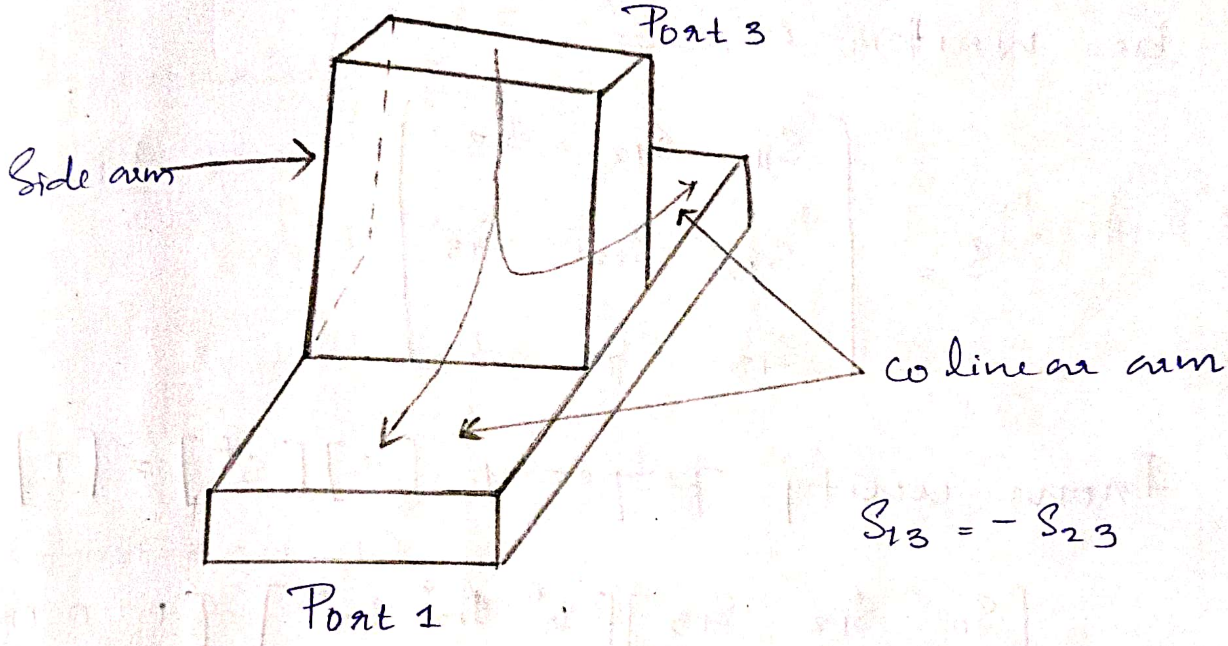
$$|S_{11}|^2 = 1/4$$

$$S_{11} = \frac{1}{2} \quad \left(\frac{1}{\sqrt{4}} = \frac{1}{2}\right)$$

$\because S_{11} = S_{22}$, $\quad S_{22} = \frac{1}{2}$ $\qquad S_{11} = -S_{12} = -1/2$

$$S = \begin{bmatrix} 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

* E-plane Tee Junction.



$S_{13} = -S_{23}$

In E-plane tee junction, the side arm is placed in the direction of electric field/ electric vector. The power distribution and therefore, the S parameter can be written as,

$$S_{13} = -S_{23}$$

As it is a three port n/w,

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

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* As port number 3 is perfectly match, $S_{33}=0$
* By using symmetry property, $S_{ij} = S_{ji}$

$$\rightarrow \quad S_{13} = S_{31}$$
$$S_{23} = S_{32}$$
$$S_{21} = S_{12}$$

With these properties, the S-matrix can be written as,

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix}$$

From unity property $[S][S^*] = [I]$

$$\Rightarrow \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* \\ S_{13}^* & -S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_1C_1 \Rightarrow \quad S_{11}S_{11}^* + S_{12}S_{12}^* + S_{13}S_{13}^* = 1$

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \quad \text{——} \; (1)$$

$R_2C_2 \rightarrow \quad S_{12}S_{12}^* + S_{22}S_{22}^* + -S_{13} \cdot -S_{13}^* = 1$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \quad \text{——} \; (2)$$

$R_3C_3 \Rightarrow \quad S_{13}S_{13}^* + -S_{13} \cdot -S_{13}^* + 0 = 1$

$$|S_{13}|^2 + |S_{13}|^2 = 1 \quad \text{——} \; (3)$$

$R_5 C_1 \rightarrow \quad S_{13} S_{11}^* + -S_{13} S_{12}^* = 0$ ——— (4)

From eqn (3);

$$|\cancel{S_{13}} \quad 2|S_{13}|^2 = 1$$

$$|S_{13}|^2 = 1/2$$

$$\underline{\underline{S_{13} = 1/\sqrt{2}}}$$

By equating (1) & (2);

(1) = (2) $\rightarrow$

$$|S_{11}|^2 + \cancel{|S_{12}|^2} + \cancel{|S_{13}|^2} = \cancel{|S_{12}|^2} + |S_{22}|^2 + \cancel{|S_{13}|^2}$$

$$|S_{11}|^2 = |S_{22}|^2$$

$$\underline{\underline{S_{11} = S_{22}}}$$

From eqn (4);

$$S_{13} S_{11}^* + -S_{13} S_{12}^* = 0$$

(4) $\rightarrow$

$$S_{13} [S_{11}^* - S_{12}^*] = 0$$

Here $S_{11}^* - S_{12}^* = 0 \qquad \because S_{13} \neq 0$

$$S_{11}^* = S_{12}^*$$

$$\underline{\underline{S_{11} = S_{12}}} \qquad [S_{11} = S_{12} = S_{22}]$$

Substitute the above parameters in (1)

(1) $\rightarrow$

$$|S_{11}|^2 + |S_{11}|^2 + |S_{13}|^2 = 1$$

$$2|S_{11}|^2 + 1/2 = 1$$

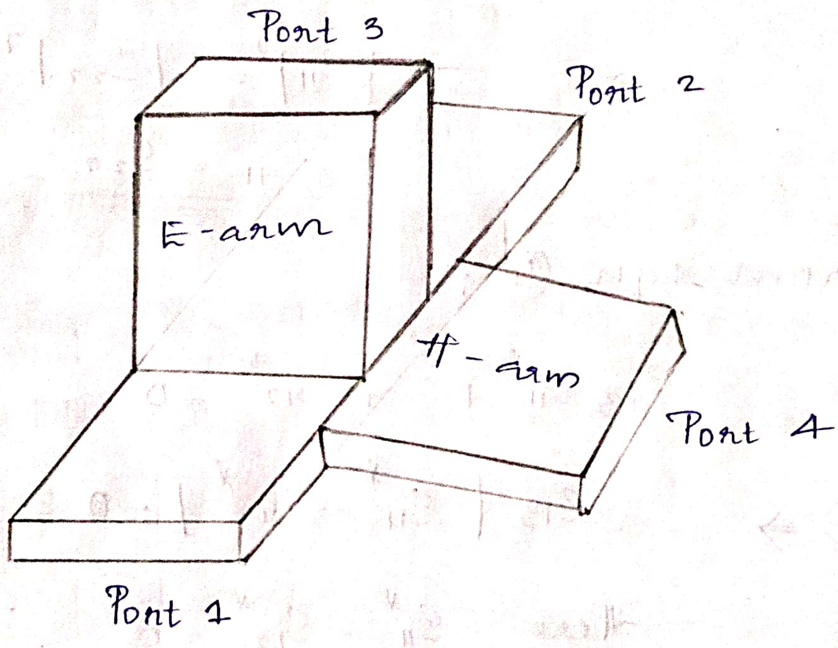
$$2|S_{11}|^2 = 1/2$$

$$|S_{11}|^2 = 1/4$$

$$S_{11} = \frac{1}{2}$$

$$\therefore \quad S = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

Magic Tee / Hybrid Tee



Here two rectangular slots are cut both along width and breadth of a long wave guide and the side arms are attached as shown in figure. Here port 1 and 2 are colinear arms; Port 3 E-arm and port 4 H-arm respectively.

As it is a four port n/w,

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

* Because of H-plane tee junction $S_{14} = S_{24}$
* Because of E-plane Tee junction $S_{13} = -S_{23}$
* As the side arms are perfectly matched

$$S_{33} = S_{44} = 0$$

* Because of geometry of junction, the i/p at port 3 cannot propagate to port 4 and vice-versa, $S_{34} = S_{43} = 0$
* By using symmetry property,

$$S_{12} = S_{21} \qquad S_{23} = S_{32}$$
$$S_{13} = S_{31} \qquad S_{34} = S_{43}$$
$$S_{14} = S_{41}$$

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & -S_{13} & S_{14} \\ S_{13} & -S_{13} & 0 & 0 \\ S_{14} & S_{14} & 0 & 0 \end{bmatrix}$$

From unity property, $[S][S^*] = [I]$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & -S_{13} & S_{14} \\ S_{13} & -S_{13} & 0 & 0 \\ S_{14} & S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* & S_{14} \\ S_{13}^* & -S_{13}^* & 0 & 0 \\ S_{14}^* & S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_1C_1 \Rightarrow |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \longrightarrow (1)$

$R_2C_2 \Rightarrow |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \longrightarrow (2)$

$R_3C_3 \Rightarrow |S_{13}|^2 + |S_{13}|^2 = 1 \longrightarrow (3)$

$R_4C_4 \Rightarrow |S_{14}|^2 + |S_{14}|^2 = 1 \longrightarrow (4)$

$(3) \Rightarrow \quad 2|S_{13}|^2 = 1$

$$|S_{13}|^2 = 1/2$$

$$S_{13} = 1/\sqrt{2}$$

$(4) \Rightarrow \quad 2|S_{14}|^2 = 1$

$$|S_{14}|^2 = 1/2$$

$$S_{14} = 1/\sqrt{2}$$

① $\Rightarrow$ $|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} + \frac{1}{2} = 1$

$$|S_{11}|^2 + |S_{12}|^2 + \cancel{1} = \cancel{1}$$

$$|S_{11}|^2 + |S_{12}|^2 = 0$$

$$\therefore \quad \underline{\underline{S_{11} = S_{12}}}$$

{as their mag are equal so no -ve}

Equating ① & ②

① = ② $\Rightarrow$

$$|S_{11}|^2 + \cancel{|S_{12}|^2} + \cancel{|S_{13}|^2} + \cancel{|S_{14}|^2} = \cancel{|S_{12}|^2} + |S_{22}|^2 + \cancel{|S_{13}|^2} + \cancel{|S_{14}|^2}$$

$$|S_{11}|^2 = |S_{22}|^2$$

$$\underline{\underline{S_{11} = S_{22}}}$$

$\therefore$ ② $\Rightarrow$

$$|S_{11}|^2 + |S_{11}|^2 + \frac{1}{2} + \frac{1}{2} = 1$$

$$2|S_{11}|^2 + \cancel{1} = \cancel{1}$$

$$2|S_{11}|^2 = 0$$

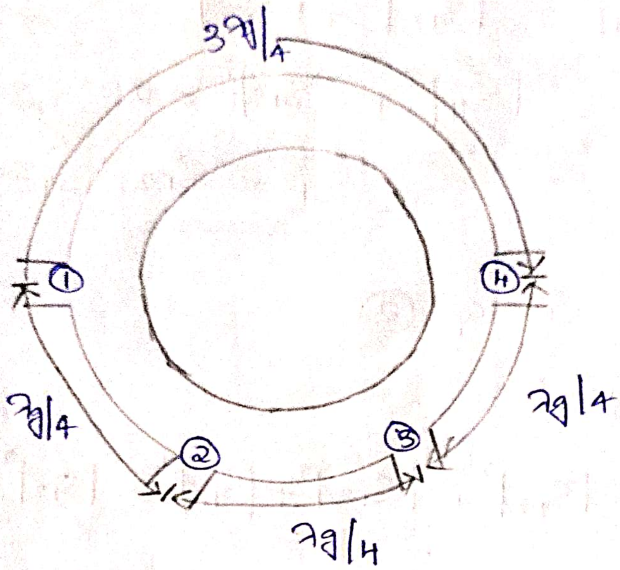
$$\underline{\underline{S_{11} = 0}}$$

$S_{11} = S_{22} = S_{12} = 0$

$S_{13} = S_{14} = 1/\sqrt{2}$

$$S = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

Rat-Race Junction / Hybrid Junction:



Here the four ports are connected in the form of an angular ring at a proper interval by means of series or parallel junctions. For proper operation, it is necessary that the mean circumference of the total race to $1.5\ \lambda g$ and each of the four ports are seperated from their neighbour by $\lambda g/4$. If the power is fed into port 1, it will divide equally into ports 2 and 4 and nothing enters to port 3. At ports 2 & 4 the powers combines inphase but at 3 cancelation occurs due to $\lambda/2$ path difference. For similar reasons the i/p applied to port 3 equally divides b/w ports 2 & 4 and the o/p at port 1 becomes zero.

Therefore,

$$S_{13} = S_{31} = 0$$
$$S_{24} = S_{42} = 0$$

As all the four ports are perfectly match, we can say,

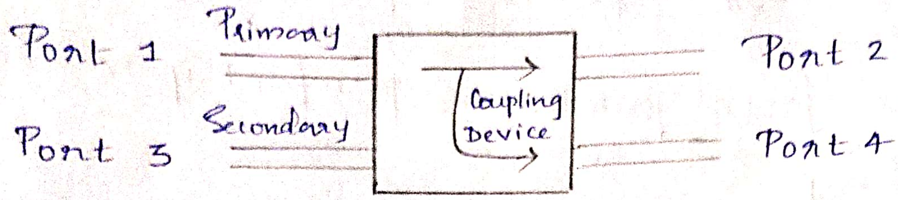
$$S_{11} = S_{22} = S_{33} = S_{44} = 0$$

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

Final S-matrix of rat-race junction:

$$S = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix}$$

Directional Couplers



It is a four port device that consist of primary wave guide (port 1-2) and secondary waveguide (port 3-4). If all the junctions are perfectly matched, then there is free transmission of power b/w port 1 and 2 and no transmission b/w port 1 & 3 and port 2 & 4. The characteristics of directional coupler can be expressed in terms of Coupling Coefficient [C] and directivity [D].

The coupling coefficient can be defined as the ratio of incident power to the forward power measured in dB.

ie.,
$$\boxed{C = 10 \log \left(\frac{P_1}{P_4}\right) \text{ dB.}}$$

Directivity is the ratio of forward power to backward power measured in dB.

ie.,
$$\boxed{D = 10 \log \left(\frac{P_4}{P_3}\right) \text{ dB}}$$

Isolation factor is the ratio of incident power P_1 to the backward power P_3

in dB.

i.e., $\boxed{\begin{aligned} I &= C + D \\ D &= 10\log\left(\frac{P_1}{P_3}\right) dB. \end{aligned}}$

The general S-matrix for a four port n/w is;

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

$S_{13} = S_{31} = S_{42} = S_{24} = 0$
[Property of directional coupler]

$S_{11} = S_{22} = S_{33} = S_{44} = 0$
[∵ all ports are perfectly matched]

$$\therefore \quad S = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix}$$

By Symmetry property

$S_{21} = S_{12}$ $\quad S_{32} = S_{23}$
$S_{14} = S_{41}$ $\quad S_{43} = S_{34}$

$$S = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix}$$

By using unity property,

$$[S][S^*] = [I]$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & 0 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_1 C_1 \Rightarrow |S_{12}|^2 + |S_{14}|^2 = 1$ ——— (1)

$R_2 C_2 \Rightarrow |S_{12}|^2 + |S_{23}|^2 = 1$ ——— (2)

$R_3 C_3 \Rightarrow |S_{23}|^2 + |S_{34}|^2 = 1$ ——— (3)

$R_4 C_4 \Rightarrow |S_{14}|^2 + |S_{34}|^2 = 1$

$R_1 C_3 \Rightarrow S_{12} S_{23}^* + S_{14} \cdot S_{34}^* = 0$ ——— (4)

Equating (1) & (2)

(1) = (2) $\Rightarrow$ $\cancel{|S_{12}|^2} + |S_{14}|^2 = \cancel{|S_{12}|^2} + |S_{23}|^2$

$$|S_{14}|^2 = |S_{23}|^2$$

$$\underline{\underline{S_{14} = S_{23}}}$$

(2) = (3) $\Rightarrow$ $|S_{12}|^2 + \cancel{|S_{23}|^2} = \cancel{|S_{23}|^2} + |S_{34}|^2$

$$|S_{12}|^2 = |S_{34}|^2$$

$$\underline{\underline{S_{12} = S_{34}}}$$

$$S = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{14} & 0 \\ 0 & S_{14} & 0 & S_{12} \\ S_{14} & 0 & S_{12} & 0 \end{bmatrix}$$

Let S_{12} be real and positive. $[S_{12} = P]$

from eqn (4);

[$S_{12} = S_{34}$ $\because$ S_{12} is +ve S_{34}^* will be +ve.]

$$S_{12} S_{23}^* + S_{14} \cdot \underbrace{S_{34}^*}_{S_{12}} = 0$$

$$S_{12} S_{23}^* + S_{14} \cdot S_{12} = 0$$

$$S_{12} \left[S_{23}^* + S_{14} \right] = 0$$

$$P \left[S_{23}^* + S_{14} \right] = 0$$

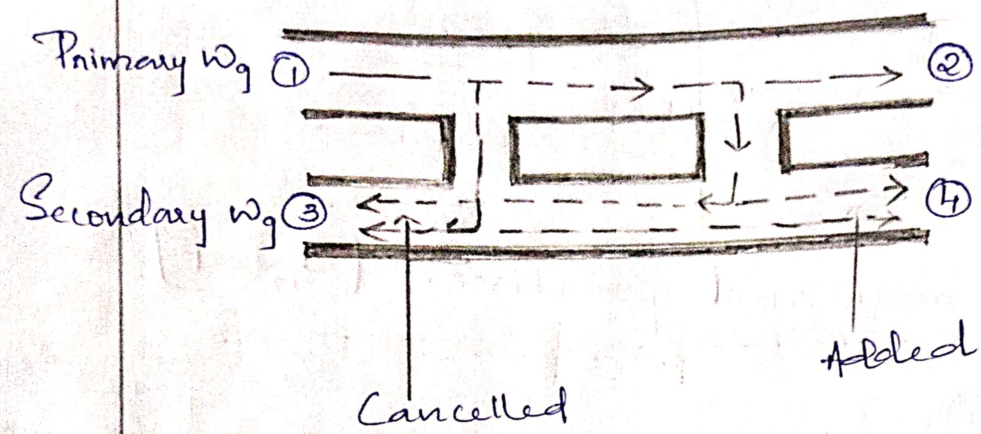
Hence $S_{23}^* + S_{14} = 0$ [for this S_{23} must be a complemed value, $\pm jq$]

$S_{23} = jq.$

$$S = \begin{bmatrix} 0 & P & 0 & jq \\ P & 0 & jq & 0 \\ 0 & jq & 0 & P \\ jq & 0 & P & 0 \end{bmatrix}$$

$S_{14} = S_{23} = jq.$

3) Two-whole directional Coupler.



A two-hole directional coupler with travelling waves is shown in figure. A fraction of energy entered in port 1 passes through the holes and radiated through secondary waveguide. The forward waves in secondary guide are in same phase and added up at port 4 whereas the backward wave in secondary waveguide are out of phase and are cancelled at port 3.

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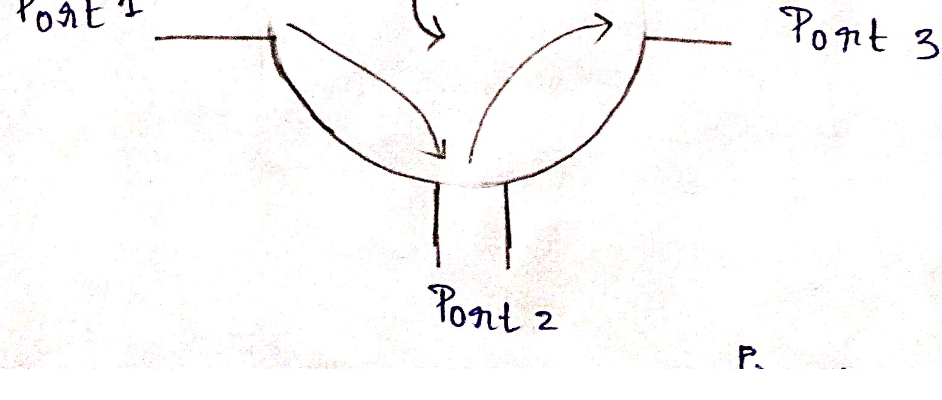
Microwave Circulator:



P.

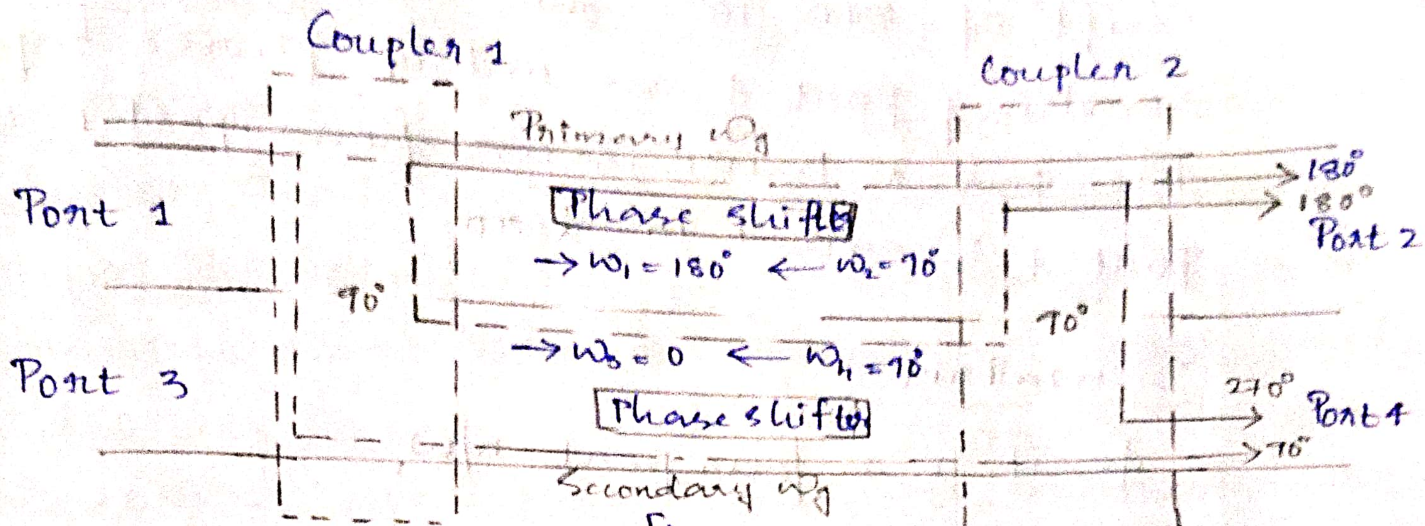


fig. 2

Microwave circulator is a multi-port wave guide junction in which the wave can flow only from n^{th} port to $(n+1)^{th}$ port in one direction. In figure 2 each of ~~the~~ 3 dB couplers introduce a phase shift of 90° and the phase shifters produces a certain amount of phase shift in the direction as indicated. When a wave incident at port 1 and split into two components by coupler 1. The wave in primary guide arrives at port 2 with a relative phase shift of 180°. The second wave propagates through two couplers & arrives at port 2 with a relative phase shift of 180°. Since the two waves reaching port 2 inphase, the power transmission obtained from port 1 to port 2.

However, the wave propagates through primary wg phase shifts and coupler 2 arrives at port 2 with a phase change of 270°. The wave travels through coupler 1 secondary wave guide arrives at port 4 with a phase shift of ~~270°~~ 90°. Since these two waves reaching port 4 are out of phase by 180° the power transmission from port 1 to port 4 is equal to zero.

S-matrix:

As it is a four port n/w,

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

* As all the four ports are perfectly matched,

$$S_{11} = S_{22} = S_{33} = S_{44} = 0$$

* From the property of circulator,

$$S_{21} = S_{32} = S_{43} = S_{14} = 1$$

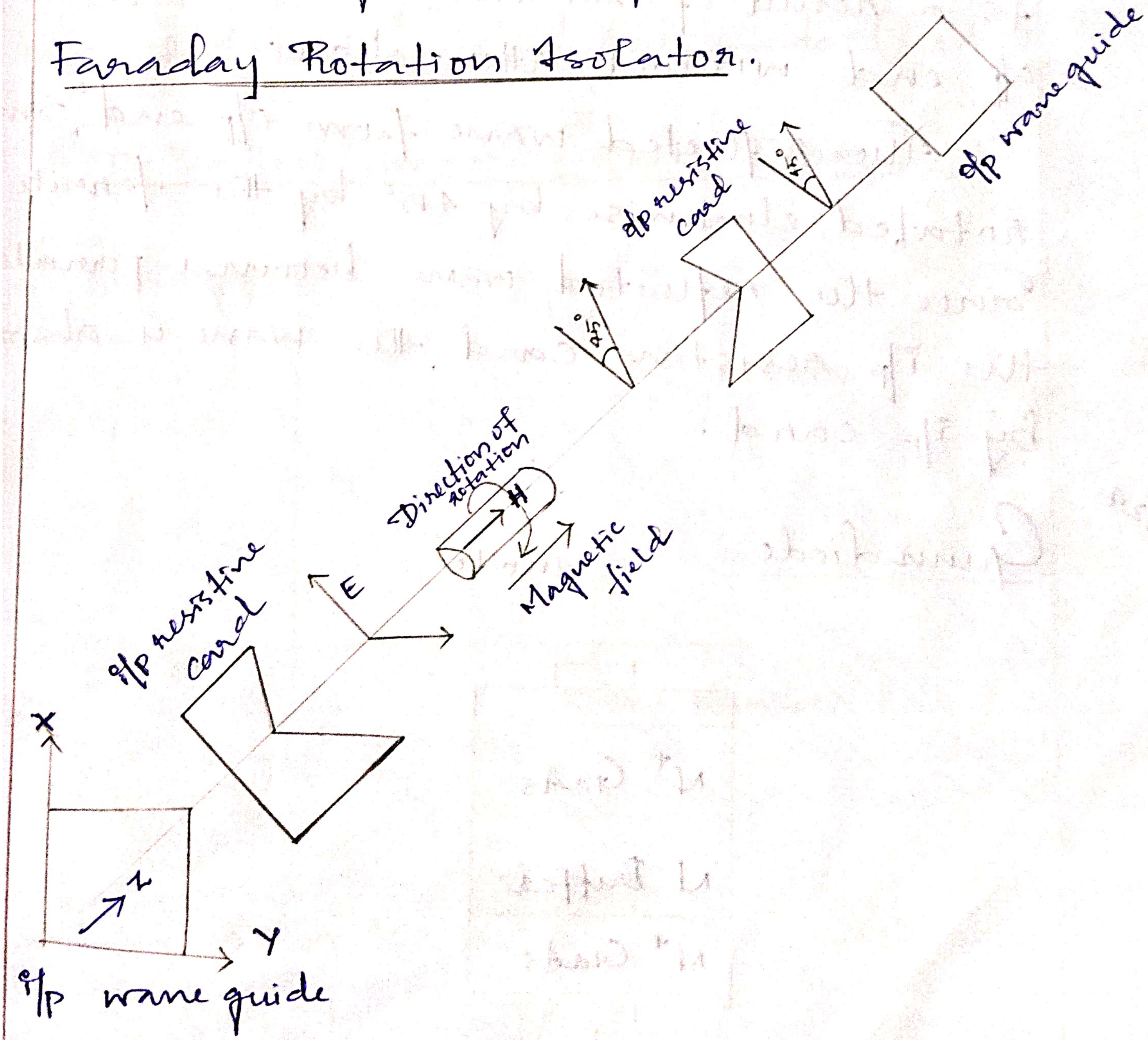
* The power received in all other ports are zero.

$$\therefore \; S = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Microwave Isolator:

An isolator is a non-reciprocal transmission device that is used to isolate one component from the reflections of other components in transmission line. An ideal isolator completely absorbs the power for propagation in one direction and provides lossless transmission in opposite direction. Therefore, isolators are also known as uniline. Isolator improves the stability of microwave generators such as klystron, magnetrons etc.

Faraday Rotation Isolator.



Here the i/p resistive card is in YZ plane, and the o/p resistive card is displaced 45° from i/p card. The magnetic field which is applied longitudinally to the ferrite rod rotates the wave plane by 45°. An i/p TE_{10} mode incident at the left end of isolator. Since TE_{10} mode, the wave is perpendicular to the i/p resistive card. Wave passes through card without attenuation. The wave in ferrite section rotated clockwise by 45° which is normal to the o/p resistive card. As a result of rotation, wave arrives at the o/p end without attenuation.

The reflected wave from o/p end is similarly rotated clockwise by 45° by the ferrite rod. Since the reflected wave becomes parallel to the i/p resistive card, the wave is absorbed by i/p card.

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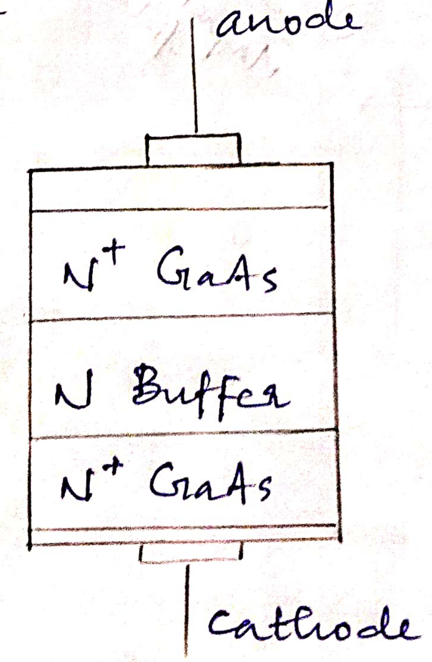
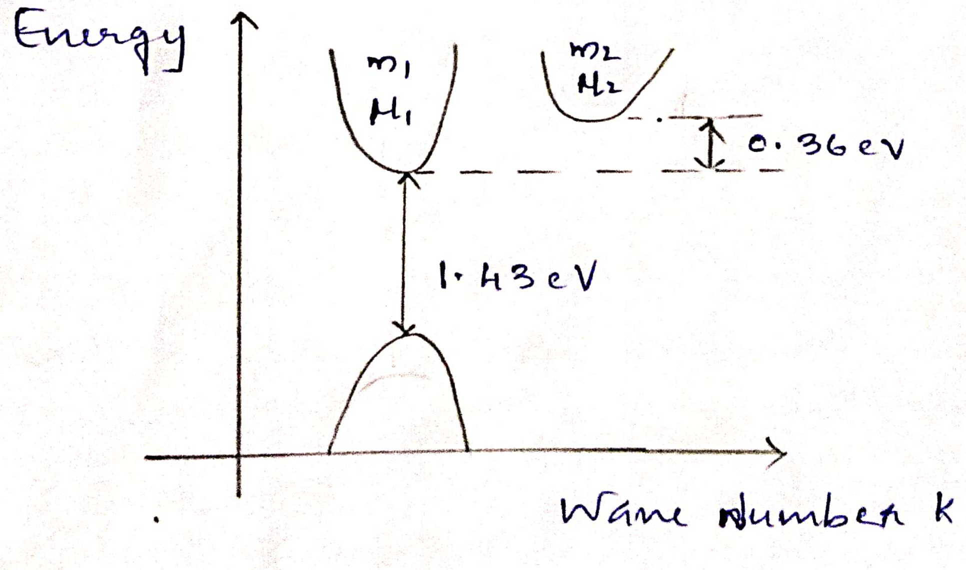
Gunn diode

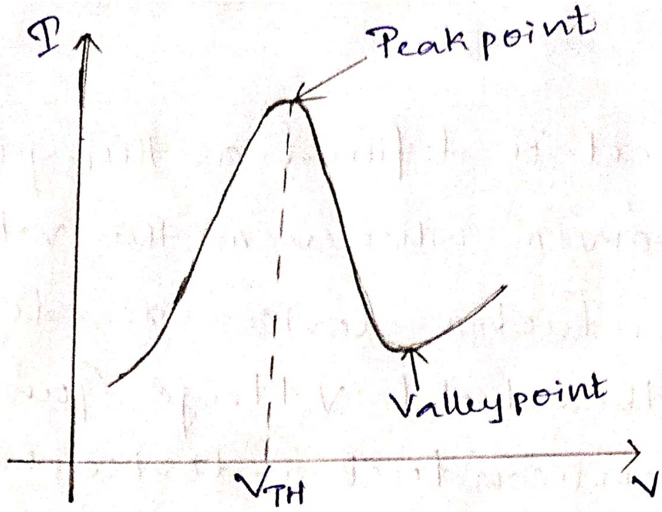


Gunn Effect:

Gunn effect is defined as the generation of microwave power whenever, the voltage applied to a Semiconductor device exceeds a critical voltage or threshold voltage. Gunn diode is a passive Semiconductor device which composes only n-type Semiconductor materials Unlike other diodes consist of pn junction. Gunn diodes can be made from materials which consist of multiple, initially-empty, closely spaced energy valleys in their conduction band like Gallium Arsenide (GaAs), InP, etc.

Two-Valley Theory [RWH Theory]





Gunn characteristics.